Geometric Considerations Of Digital Twinning The Trajectories Of Unmanned Aerial Vehicles

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Abstract. Unmanned aerial vehicles (UAV) have recently emerged as a valuable technological asset tailored for diverse assignments ranging from area or target surveillance and psychological operators to critical equipment pickup or delivery, while maintaining a low observability profile. In addition, they can operate in swarms, multiplying their efficiency under appropriate coordination schemes. Their capabilities are important in urban metroplexes where the smart deployment of small, light, and highly mobile parties acting and reacting rapidly in three dimensions based on local intelligence is crucial. Consequently, the location and the trajectory of a UAV can be mined for patterns. The latter can be facilitated by digital twinning, which requires the translation of geometric quantities to computational ones. These considerations are enumerated along with a detailed discussion.

Keywords: Digital twinning · Simulation · Unmanned aerial vehicles · UAV · Specifications · UAV trajectories · Geometric analytics · Geometric twinning · Noise models · Julian code.

1 Introduction

Implementing and deploying a simulation is a major step in contemporary operations across a broad spectrum of financial and technological fields. Digital twinning of physical systems is the next evolutionary step in the field of simulation and allows the safe and quick assessment of a potentially complex existing system with a myriad of interactions with its environment. This is accomplished by

physical characteristics . . .

Emphasis in placed on the digital twinning of the quantum gates per se and not any actual quantum computation in the same way the digital twinning of a complex physical system seeks to replicate its inner workings at various resolution levels, starting from a rather abstract description of the overall system and right down to its elementary constituent components, and not to evaluate how these components can be combined for a particular purpose. As an example from another field, consider the digital twinning of an unmanned aerial vehicle

(UAV). running a distributed coordination protocol aims at evaluating how the latter works and its efficiency and not at

Moreover, the role of machine learning operating on geometric attributes to draw conclusions regarding the UAV trajectories is also explored.

The primary research objective of this conference paper is a systematic outline of the geometric considerations regarding the digital twinning of the UAV trajectories. These include the discretization of the aerial space, modelling the velocity and acceleration, evaluating the effect of noisy measurements, and discovering circular segments. As a secondary objective, the ...

The remainder of this work is structured as follows. ... 2 ... Capital bold letters denote matrices, small bold vectors, and small scalars. Unless explicitly stated otherwise or made clear from context, these quantities are to be assumed to be complex. Acronyms are explained the first time they are encountered in text. Finally, the notation of this work is summarized in table 1.

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Table 1. Notation synopsis.

Symbol	**Meaning**	**First in**
$\triangleq$	Definition or equality by definition	Eq. (1)
$\lvert\cdot\rvert$	Absolute value or complex magnitude	Eq.
$\lVert\cdot\rVert$	Vector or matrix norm	Eq. (3)
$\mathbf{I}_n$	$n \times n$ identity matrix	Eq.
$\{s_1, \ldots, s_n\}$	Set with elements $s_1, \ldots, s_n$	Eq. (1)

2 Previous Work

... [6] ... [4] ... simulating take off during a wide array of wind conditions [1], ...

ML has numerous applications to ... Reinforcement learning (RL) in conjuction with digital twinning [9] has been proposed as a reliable and adaptive way to control the motion of UAV swarms [8]. In this context the digital twin can act as a trainer to physical UAVs [7] ... Self organizing maps (SOMs) [2] rely heavily on topological properties ... Trajectories can be clustered with [3] ... Objects from UAV ... local curvature is approximated in the context of self organizing maps in [5] ...

Digital twinning complexity ... Simulation in polynomial time ... theorem ...

3 UAV Trajectories

3.1 Notions

In this section the basic notions of UAV trajectories and how these can be translated to digital twinning are explained. Given that trajectories take place in a three dimensional space, it follows immediately that geometry plays an instrumental role. To this end, ML architectures taking into consideration spatial properties as well as physics-based models will be quite useful.

The location vector $\mathbf{r}$ denotes at any given time space the UAV location as shown in equation (1). The coordinates can be absolute or relative to a reference point and their precision relies heavily on the UAV hardware, whereas their as well as any geolocation and triangulation methods employed to keep track of the position of a UAV in real time.

$$\mathbf{r}[k] \triangleq \begin{bmatrix} x[k] & y[k] & z[k] \end{bmatrix}^T \tag{1}$$

The first dimension is discretized with resolution h_x from a reference level x_0, typically the ground, as a set of equidistant points as shown in equation (2). As a convention, the space resolution is the same for each dimension and typically relies on the UAV sensors. Moreover, when there are natural obstacles or no-fly zones, the space resolution can be adjusted or special points can be marked as such in order to avoid violating local restrictions. In the former case, the definitions of (3) and (4) still work and yield reasonable results at the boundary points, perhaps with a variable spatial resolution scheme involving control points.

$$X \triangleq \{x_0 + kh_x\}, \quad x_0 \,\leq\, X \,\leq\, x_0 + (n-1)h_x \tag{2}$$

Based on the above, the velocity of a UAV defined as in equation (3), is the first discrete difference of the location vector. Also, since velocity is a vector, its direction is determined by the angle formed by the lines spanned by the old and new location vectors. The angle between them is a measure of how abruptly velocity changes in terms of direction. The sharper the direction change, the more power needs to be diverted to the UAV engines or rotor blades, which in turn dictates more delicate power and control circuits. Moreover, the UAV endures higher stress and potentially dissipates more heat to the environment.

$$\mathbf{v}[k] \triangleq \begin{cases} \left\|\mathbf{r}[k] - \mathbf{r}[k-1]\right\|_2, & \text{magnitude } |\mathbf{v}[k]| \\[2ex] \arccos\left(\dfrac{\mathbf{r}[k]^T\mathbf{r}[k-1]}{\left\|\mathbf{r}[k]\right\|_2 \left\|\mathbf{r}[k-1]\right\|_2}\right), & \text{angle } \angle\mathbf{v}[k] \end{cases} \tag{3}$$

Along a similar line of reasoning the acceleration (4) can be defined as the discrete second derivative of the location vector. Regarding the acceleration angle

$\angle \mathbf{a}[k]$, it is also a measure of location variability similarly to $\angle \mathbf{v}[k]$.

$$\mathbf{a}[k] \triangleq \begin{cases} \frac{1}{2} \left\| \mathbf{r}[k] - 2\mathbf{r}[k-1] + \mathbf{r}[k-2] \right\|_2 , & \text{magnitude } |\mathbf{a}[k]| \\ \arccos \left(\frac{\mathbf{r}[k]^T \mathbf{r}[k-2]}{\left\|\mathbf{r}[k]\right\|_2 \left\|\mathbf{r}[k-2]\right\|_2} \right) & \text{angle } \angle \mathbf{a}[k] \end{cases} \tag{4}$$

Concerning the acceleration angle ... higher granularity ...

$$\vartheta_1[k] \triangleq \frac{}{\| \|_2 \, \| \|_2} \quad \text{and} \quad \vartheta_2[k] \triangleq \frac{}{\| \|_2 \, \| \|_2} \tag{5}$$

Angles ϑ_1 and ϑ_2 offer ...

3.2 Angle computation stability

The computation of both ... requires ... The numerical stability of ... The inner product is backwards stable in the IEEE floating point arithmetic model and so is the squared norm, being the inner product of a vector with itself for real vectors or with its conjugate for complex ones.

3.3 Circular segments

Detecting angles ...

Observe that when the velocity vector has a constant magnitude γ_0, then its derivative is constantly perpendicular as can be seen from (6). This is useful in detecting circular motion or circular segments in a trajectory by computing the angle between the velocity and the acceleration vectors. In equation (6) the two terms in the third segment following the differentiation of the squared norm are the same representing the inner product of vector $\mathbf{v}$ with its gradient $\nabla \mathbf{v}$. In the context of this work the latter is approximated by the acceleration vector.

$$\|\mathbf{v}\|_2^2 \;=\; \gamma_0^2 \;\Rightarrow\; \nabla \|\mathbf{v}\|_2^2 \;=\; 0 \;\Leftrightarrow\; \mathbf{v}^T \nabla \mathbf{v} + \nabla \mathbf{v}^T \mathbf{v} \;=\; 0 \;\Leftrightarrow\; \mathbf{v}^T \nabla \mathbf{v} \;=\; 0 \tag{6}$$

Another way to track ... local curvature ... For a continuous function the curvature κ is defined as in equation (7).

$$\kappa \triangleq \frac{f}{f} \tag{7}$$

4 Noisy Measurements

As the velocity of the UAV is estimated, it makes sense to consider that its measurements are corrupted by noise. The most common scenario is to consider the effects of additive white Gaussian noise (AWGN) which probabilistically corresponds to the contribution of many factors of comparable strength such as the thermal noise in electronics.

$$\mathbf{v}_m[k] \triangleq \mathbf{v}[k] + \mathbf{w}_a[k] \tag{8}$$

Recall that ...

$$\mathbf{w}_a[k] \;=\; \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(w-\mu)^2}{2\sigma^2}\right) \tag{9}$$

Another noise model ...

$$\mathbf{v}_m[k] \triangleq \mathbf{v}[k]\mathbf{w}_p[k] \tag{10}$$

...

$$\mathbf{w}_p[k] \triangleq \alpha_0 k^{-\gamma_0} \tag{11}$$

The UAV trajectories can be mined in order to find patterns regarding the environment of the UAV iteself in case the latter is controlled by another party. ...

Location correction and UAV guidance require feedback ...

As a concrete example ...

5 Digital Twins

6 Functools Module Overview

The *functools* module contains tools enhancing code functionality ... The functools module allows the construction of partial objects and functions. ...

Table 2. Functools Functionality And Python Concepts

Name	**Description**	**Shown in**
map	Application of a function to iterables	
filter	Selection of data from iterables	
reduce		
partial		
@lru_cache		
@totalordering		
yield		
lambda		
callable		
iterable		

As was the case with phase gates, the implementation of rotation matrices in Python can be done with a class. ...

Table 3. Functions To Be Implemented

Function	**Meaning**
__str__	
__repr__	
__add__	
__sub__	
__mult__	
__truediv__	
__eq__	
__lt__	

... Moreover, since a rotation gate is uniquely determined by its angle θ, it makes sense to be able to create new such gates with a (sub)multiple of a given angle. In fact, it is common in quantum communications to create gates whose angle is half or double that of another such gate in the same circuit. Thus, the operators of multiplication and division have been also overloaded to offer this functionality through implementing __mult__ and __truediv__ respectively. The latter is the method for implementing division in Python 3, in contrast to the __div__ method which corresponds to division in Python 2 and it is not supported in Python 3. In table 3 ...

7 Conclusions And Future Work

In this work the functionality of the Python module *functools* is overviewed. ...

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