Wavelet-based Mammographic Enhancement

H. Stefanou[1], S. Kakouros[2], D. Cavouras[3], M.Wallace[4]

[1] Department of Computer Science,
University of Crete, Greece
[2] Department of Informatics and Telecommunications,
National and Kapodistrian University of Athens, Greece
[3] Department of Medical Instrumentation Technology,
Technological Educational Institution of Athens, Greece
[4] Image, Video and Multimedia Systems Laboratory
National Technical University of Athens, Greece
e-mail: hstef@csd.uoc.gr

Abstract

In mammography the interesting characteristics of an image are malignant masses, microcalcifications and skin thickening, of which the last two are said to be indirect signs of malignancy. The mammograms, as normally viewed, display a small percentage of the information they detect and that is due to the minor difference in x-ray attenuation between normal glandular tissues and malignant disease. This makes the detection of small malignancies difficult. The digital medical image processing uses denoising and image enhancement techniques so as to reveal any tumors that may not be obvious and help the oncologist decide. In this paper we employ two different methods of image enhancement and compare them in order to decide which one provides better results in each mammogram. The first method is called adaptive enhancement algorithm and it measures the correlation between wavelet coefficients in successive levels of the analysis so as to decide if the coefficient derives from noise or not and then uses non-linear mapping for the enhancement. The second one, which we call typical method of enhancement, is a method which uses thresholding for the denoising step, non-linear mapping in order to enhance the image and finally filtering to deblur and sharpen the mammogram.

Key-words

Wavelets, À trous Algorithm, Adaptive Enhancement Algorithm, Thresholding, Non-linear Mapping Functions, Image Sharpening.

1 Introduction

From statistical surveys which have taken place the last years, we have, unfortunately, confronted the fact that breast cancer affects one every nine women in the world. In Europe, it is the major reason for women mortality who age between 35 and 55. As far as the mortality caused by this type of cancer in Greece is concerned, an increase of 53.27% has been noticed since 1970, while 4000 new incidents occur every year (Greece Now). The early detection of breast cancer is clearly a key ingredient of any strategy designed to reduce breast cancer mortality. Due to the large numbers of possible patients and their uniform distribution in urban and rural environments, in person precautionary periodic examination of the complete population by physicians seems to be impossible. Thus, research is gradually turning towards the development of methodologies and tools that can allow this process to happen in an automated or networked environment. The goal is not necessarily to provide perfect estimations in all cases; rough evaluations but without any false negatives are enough to filter out a large portion of the population and make precautionary examinations more easily tractable. In this process, computerized systems will be required to examine medical tests, such as mammograms, and identify the elements a physician would also look for. As a first step, the system needs to be able to discriminate between artificial noise and actual signal, before continuing to make an initial evaluation.

Many studies have been carried out in order to satisfy this purpose. The proposed digital processing techniques have been applied previously to mammography. The focus of these investigations has been to enhance mammographic features while reducing the enhancement of noise. This is a complicated matter because it is not easy to discriminate the noise from the features of the initial image. The study should keep the balance between the enhancement and the denoising of the image so as not to lose any important feature but also make more distinct the mammogram itself. That is what we tried to do in both our enhancement methods by selecting the most effective parameters.

In this paper, the à trous algorithm, an undecimated wavelet transform, will be introduced and two methods of mammograms' enhancement will follow. The first method, the adaptive enhancement algorithm, measures the correlation between wavelet coefficients in successive levels of the analysis so as to decide if the coefficient derives from noise or not and then makes use of non-linear mapping functions for the enhancement. The second one uses thresholding for the denoising step, non-linear mapping functions in order to enhance the image and finally high-pass spatial filtering to sharpen the mammogram.

2 Undecimated Wavelet Transform – The À Trous Algorithm

Decimation, which is used in the discrete wavelet transform, generates a shift variant transformation and this problem has to be overcome. Some undecimated wavelet transforms have been introduced as a solution, i.e. the à trous algorithm (Shensa, 1992). The idea that lies beneath this algorithm is to upsample the low-pass filter g and spread it so as to provide space in which to put the interpolated values. Then, a filter h is applied which leaves the even points fixed and interpolates to get the odd points.

Definition 1 The low-pass filter h is said to be an à trous filter if it satisfies

(2.1) $$h_{2n} = \delta(n) / \sqrt{2}.$$

The result of the entire interpolation, as shown in Fig. 1, is thus

(2.2) $$[h * Dg]_k = [H \cdot Dg]_k = \sum_n h_{k-2n}\psi(n) \approx \frac{1}{\sqrt{2}}\psi(k/2).$$

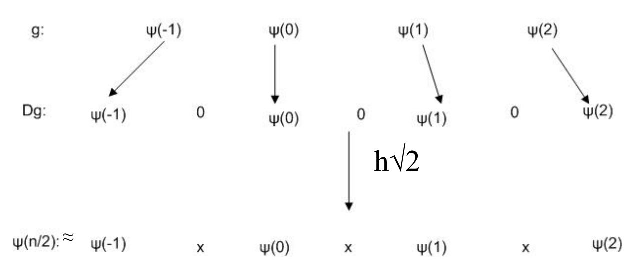

Figure 1 - Dilation and interpolation of a function ψ(t).

The à trous algorithm is described by the following equations,

$$f^{i+1} = \Lambda(h * f^{i}) \quad \text{and} \quad \tilde{f}^{\,i} = g * h \tag{2.3}$$

where $\Lambda_{k,m}=\delta(2k-m)=\delta_{2k,m}$ is the decimation operator.

The above relations are not shift invariant and the undecimated version of the algorithm enters now. Let T_m be the operation of translation by m:

$$(T_m f)_k \equiv f_{k-m} \tag{2.4}$$

We are able to see that $\tilde{f}^{\,i}$ is translation variant but if we replace m with $2^i m$ we get

$$[\tilde{f}^{\,i}(T_{2^i m} f^0)]_k = [\tilde{f}^{\,i}(f^0)]_{k-m} \tag{2.5}$$

thus, translating f^0 by $2^i m$ translates octave i by m.

Definition 2 The undecimated $\bar{f}$ in terms of the decimated transform $\tilde{f}$ is

$$\tilde{f}^{\,i}_k \equiv [\bar{f}^{\,i}(f^0)]_k \equiv [\tilde{f}^{\,i}(T_{-k} f^0)]_0 . \tag{2.6}$$

The equations that describe the undecimated à trous algorithm are

$$f^{i+1} = (D^i h) * f^{i} \quad \text{and} \quad \bar{f}^{\,i} = (D^i g) * f^{i} . \tag{2.7}$$

3 Adaptive Enhancement Algorithm

The adaptive enhancement algorithm (Brown, 2000) uses the correlation between wavelet coefficients at the same spatial position in successive resolution levels in order to discriminate between coefficients arising from noise within the image and those arising from signal features. Noise is normally spatially localized so in the wavelet domain coefficients due to noise are normally weakly correlated with the corresponding ones on successor planes.

The algorithm consists of the following steps:
Implement a redundant wavelet transform on the image to be enhanced.
  For each wavelet plane except the last
  For each position in the image domain
    compute the correlation with the corresponding coefficient on the successor plane.
    From the correlation derive a value which measures the 'evidence' that the wavelet coefficient is caused by noise.
    Use this evidence value to determine what gain to use.
    Select a mapping function determined by this gain and apply it to the coefficient.
Finally implement an inverse wavelet transform to construct the enhanced image.

3.1 Computing the Evidence of Noise from the Wavelet Correlation

Given a wavelet coefficient set at resolution n, and its successor in the next highest resolution level (n +1), for each position we get :

$$Corr_n = W_n * W_{n+1} \tag{3.1.}$$

We must now scale the set of correlation values because on average wavelet coefficient magnitudes decline with increasing resolution level. We compute:

$$P_{corr} = \Sigma\,(Corr_n * Corr_n) \quad \text{: total power in the correlation values} \tag{3.2.}$$

$$P_w = \Sigma\,(W_n * W_n) \quad \text{: total power in the } n^{th} \text{ coefficient set} \tag{3.3.}$$

The summations are carried out over the full set of wavelet coefficients. The scaled correlation values are:

(3.4.) $$Corr2_n = Corr_n * \sqrt{(P_w / P_{corr})}$$

and the minimum absolute value within the set of scaled correlation values is found:

(3.5.) $$Corr2min = \text{minimum } \{|Corr2_n|\}$$

This value is treated as indicating definite noise. For each position a proper evidence value, as shown in Table 1, is assigned.

```
if (abs(Corr2n) ≥ abs(Wn))
        E = 0
else if (abs(Corr2n) = Corr2min)
        E = 1
else E = (abs(Wn) - abs(Corr2n)) / (abs(Wn) - Corr2min)
```

Table 1

3.2 Adapting the Enhancement Process

For each wavelet coefficient the evidence value determines which mapping function to apply. The possible mapping functions fall within an envelope of mapping functions (Fig.2).

When the evidence suggests that the wavelet coefficient is definitely due to noise then the mapping function applied is defined by the light solid line. For wavelet coefficient magnitudes smaller than $|T_1|$ the gain is zero. Above $|T_2|$ the gain is unity. For coefficients whose absolute magnitude falls between T_1 and T_2, the gain rises uniformly from 0 to 1. On the other hand, if the wavelet coefficient arises from signal features, then the mapping function to be applied corresponds to the heavy line. In this case, a maximum gain G_{max} is applied if the wavelet coefficient value is smaller than $|T_1|$. If the absolute value of the wavelet coefficient falls between T_1 and T_2 then the gain falls uniformly to unity, and beyond T_2 remains at that value so as to minimise the amplification of already strong edge features.

In the case of an evidence value which is not decisive, a gain value between zero and G_{max} is used. If the evidence value is above some upper limit then the gain is set to zero. Alternatively if the evidence value is below a lower limit the gain is set to G_{max}, the maximum gain to be applied to this wavelet coefficient set. Linear scaling is applied when the evidence falls between E_{lower} and E_{upper} (Fig. 3).

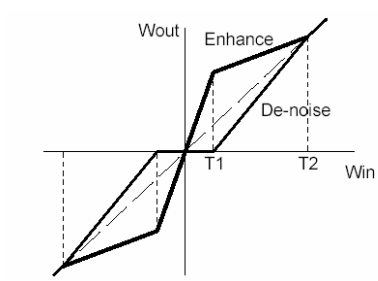

Figure 2 – Envelope of wavelet mapping function used by the adaptive algorithm

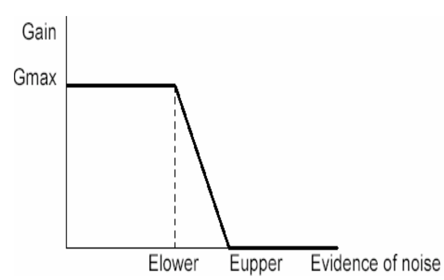

Figure 3 - Deriving gain from evidence of noise

4 Typical Method of Enhancement

The typical method of enhancement consists of three steps. The first one is denoising using thresholds, the second one is enhancement via non-linear mapping functions and finally sharpening of the image with the use of high-pass spatial filtering (Stefanou et al., 2005).

4.1 Denoising

The coefficients arising from noise are characterized by high frequency so most denoising techniques are methods of low-pass filtering in which channels of higher frequencies are cut off while channels of lower ones are enhanced. A simple way of denoising is thresholding where the wavelet coefficients whose magnitude are below a given value – the threshold – are set to zero. In this paper we use different values of thresholds at each wavelet level, which correspond to percentages of the set of coefficients.

4.2 Enhancement

Linear mapping functions, where all wavelet coefficients are multiplied by a certain gain value, tend to enhance sharp edges. If a mammogram which contains a single obvious (high intensity) microcalcification, for example, is enhanced by a linear operator it will result in gross rescaling within the available dynamic range of a display. This problem is solved by a simple non-linear method.

A non-linear function is that shown in Fig. 4 and given by (4.1) where K_1, $K_2>1$ so that the wavelet coefficients are enhanced. As observed, the function comprises of two linear parts with different slopes which correspond to each gain. If $K_1>K_2$ coefficients with small magnitudes are amplified, thus weak features of the image are enhanced, while if $K_1<K_2$ sharp features are enhanced. Another function is the one appeared in Fig. 5 and described by (4.2). This function has two thresholds, therefore, categorizes the coefficients in three groups. The last function is described by (4.3) and shown in Fig. 6. The coefficients with magnitude close to zero are almost totally suppressed while those with larger magnitude are amplified and, thus, enhanced. A linear function is applied to the coefficients with magnitude above T.

$$E(x)=\begin{cases} K_2x-K_1T, & if \quad x<-T \\ K_1x, & if \quad |x|\leq T \\ K_2x+K_1T, & if \quad x>T \end{cases} \tag{4.1}$$

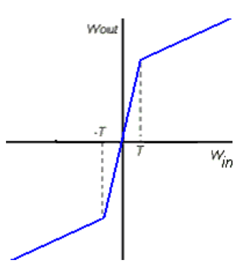

Figure 4 – Bilinear mapping function

$$E(x)=\begin{cases} K_1x-K_2T_2, & if \quad x>-T_2 \\ K_2x-K_1T_1, & if \quad -T_1<x<-T_2 \\ K_1x, & if \quad |x|\leq T_1 \\ K_2x+K_1T_1, & if \quad T_1<x<T_2 \\ K_1x+K_2T_2 & if \quad x>T_2 \end{cases} \tag{4.2}$$

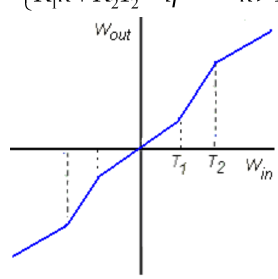

Figure 5 – Twin-threshold mapping function

$$E(x)=\begin{cases} K_2x-K_1T^2, & if \quad x<-T \\ K_1x^2, & if \quad |x|\leq T \\ K_2x+K_1T^2, & if \quad x>T \end{cases} \tag{4.3}$$

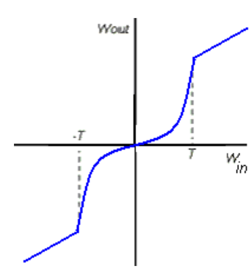

Figure 6 – Broken mapping function

4.3 Restoration

Restoring an image is the way of eliminating the degradation that the image sustains during the processing. A type of degradation is the blurring which evokes at the low-pass filtering used in the denoising.

The filtering in the spatial coordinates range of the image, which deblurs the image, is achieved by a convolution with a n×n matrix:

$$y(i,j) = \sum_{k=i-w}^{i+w} \sum_{l=j-w}^{j+w} f(k,l)h(i-k, j-l) \quad (4.4)$$

where f: input image, h: filter function, y: output image.

The basic high-pass spatial filtering is used to sharpen the image by amplifying the high frequencies coordinates. The filter should have positive coefficients near the centre and negative ones radially, while the sum of the filter coefficients should be zero in order to recognize an area with no edges.

5 Conclusion - Results

For a mammogram processed with the typical method at different levels of wavelet analysis we have chosen the parameters and function which produced the best results (Fig.7).

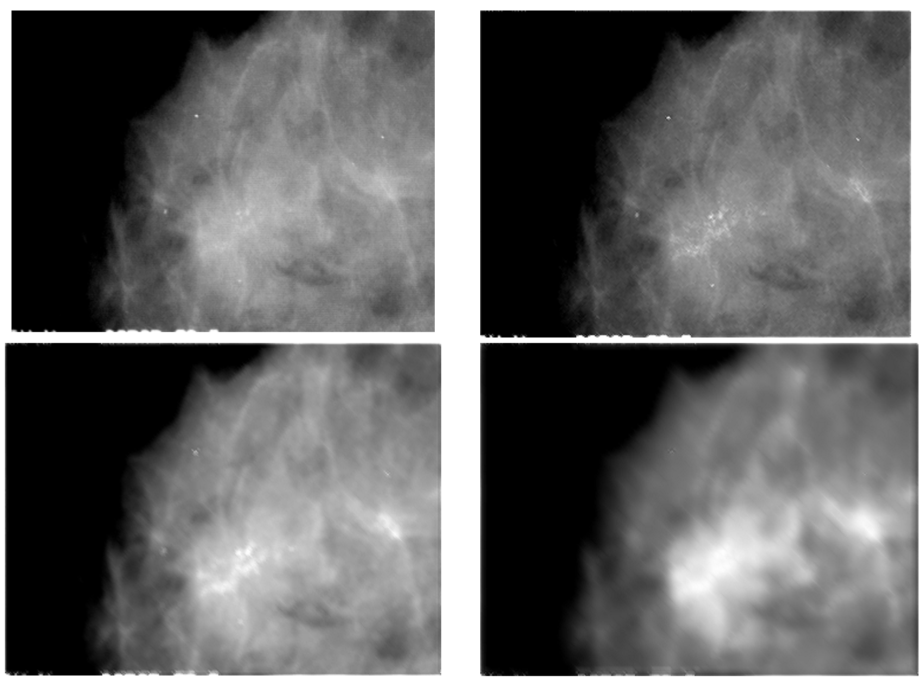

Figure 7 - Initial image (up left), output image of first level of analysis(up right), second level of analysis(down left), third level of analysis(down right).

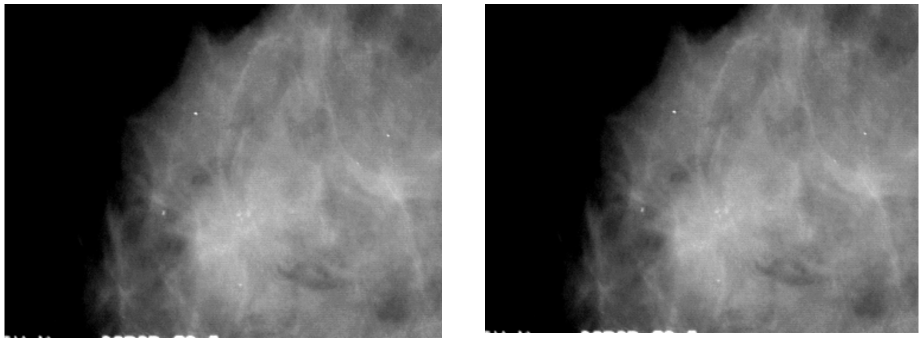

Figure 8 – Initial image (left) and output image (right).

We observe that the typical method produces better results when the analysis is made at the first level because it introduces blurring with the increase of the analysis levels. The adaptive algorithm is better when the analysis is made at higher levels because it does not blur the image. When the previous image is processed with the adaptive enhancement algorithm at the second level of the wavelet analysis the following result is produced (Fig. 8).

Another comparison is presented below.

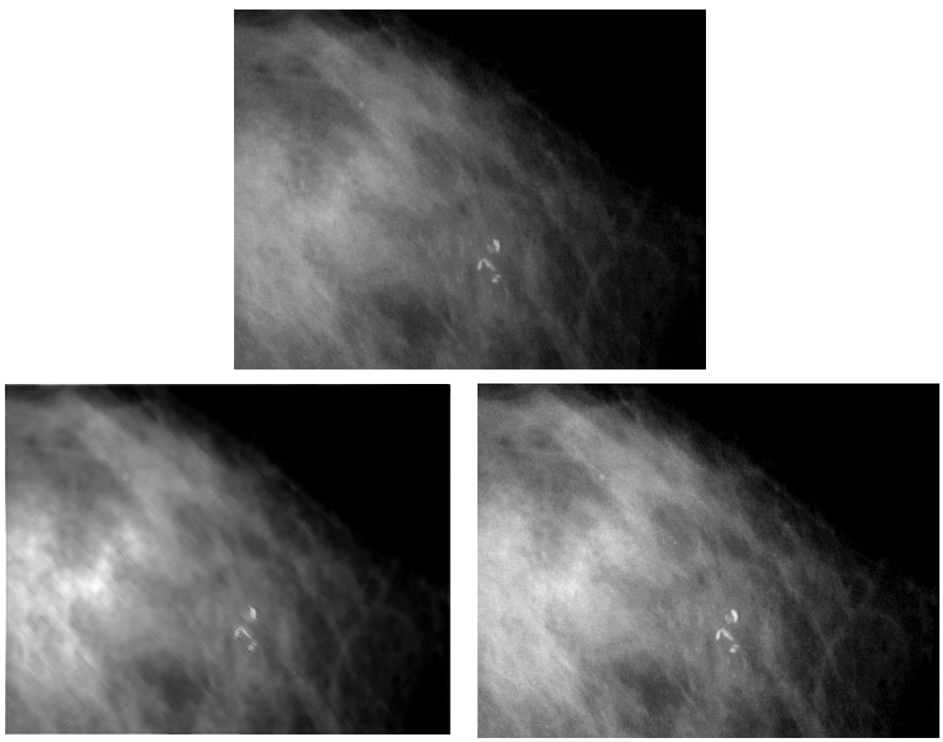

Figure 9 – Initial image (up), image processed by typical method at 2nd level (down-left) and image processed by adaptive algorithm (down-right)

Some other results of the two methods are shown in the next figures. The enhancement caused by the typical method at the first level of the analysis is obvious in Fig.10, while the use of the adaptive method at the second level produces not enough enhanced results (Fig. 11).

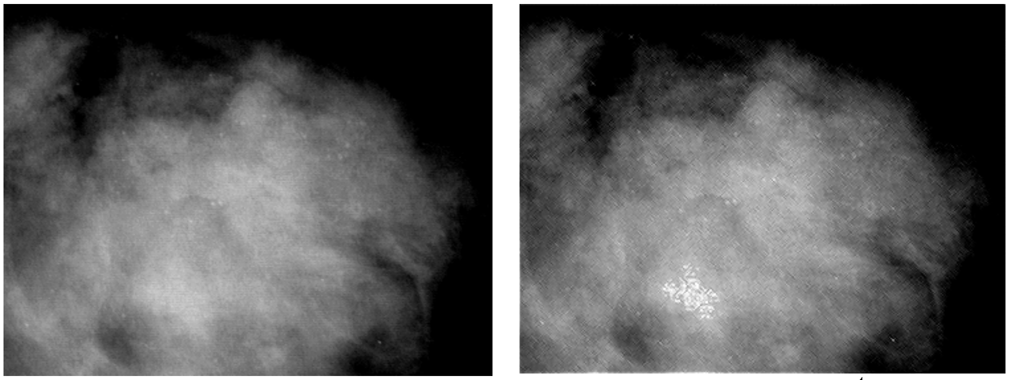

Figure 10 - Initial image (left) and processed by typical method at 1st level (right)

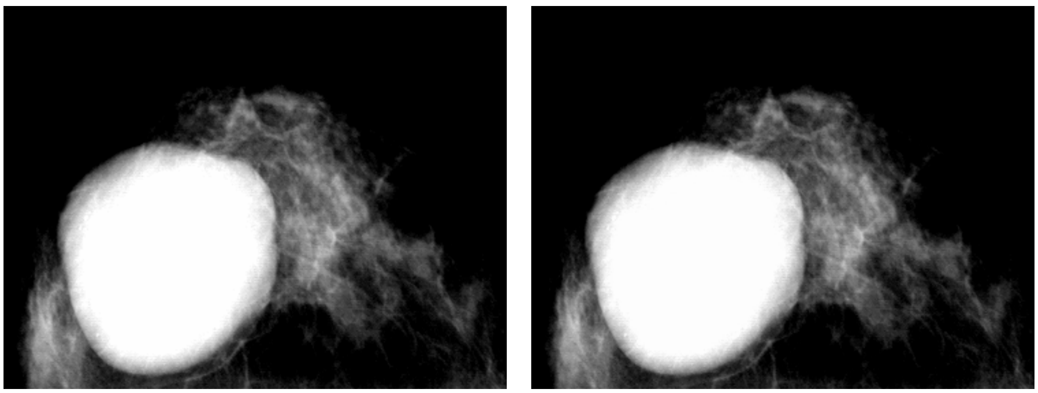

Figure 11 - Initial image (left) and processed by adaptive algorithm at 2nd level (right)

6 References

Brown, T.J. (2000), “An Adaptive Strategy for Wavelet Based Image Enhancement”, *IMVIP Proc.: Irish Machine Vision and Image Processing Conference*, pp. 67-81.

Greece Now Web Site, “Campaigning vs ignorance, Apathy, neglect and fear: key players in breast cancer epidemic in Greece”, http://www.greece.gr/LIFE/Lifestyle/campaigningvsignorance.stm, (accessed 27 January 2005).

Maglogiannis, I., Pavlopoulos, S., Koumpouros, I., Koutsouris, D. (2000) “A computer based analysis system for dermatological images”, 2nd European Symposium on Biomedical Engineering and Medical Physics, 6-8 October 2000, Patras, Greece.

Misiti, M., Misiti, Y., Oppenheim, G., Poggi, J.M., “Wavelet Toolbox For Use with MATLAB: User’s Guide Version 2”, The MathWorks.

Shensa, M. J. (1992), “The Discrete Wavelet Transform: Wedding the À Trous and Mallat Algorithms”, *IEEE Transactions on Signal Processing*, vol.40, no.10, pp.2464-2482.

Stefanou, H., Kakouros, S., Theodoridis, S., Cavouras, D. (2005), “Mammogram Analysis Using Wavelets”, Unpublished thesis.