Extending Fuzzy Description Logics for the Semantic Web[(*)]

Giorgos Stoilos[1] and Giorgos Stamou[1]

Department of Electrical and Computer Engineering, National Technical University of Athens, Zographou 15780, Greece

Abstract. Fuzzy Description Logics (Fuzzy DLs) and fuzzy OWL have been proposed as languages able to represent and reason about imprecise and vague knowledge. Such extensions have gained considerable attention the last couple of years since on the one hand they are pivotal for applications that are inherently imprecise, like multimedia analysis and retrieval, geospatial applications and more, while on the other hand they can be applied to Semantic Web applications, like querying with preferences, modelling levels of trust and proof and more. In the current paper we extend the current state-of-the-art on fuzzy extensions to Semantic Web languages by presenting the syntax and semantics of the fuzzy-$\mathcal{SROIQ}$ DL as well as the abstract, XML syntax and semantics of a fuzzy extension to OWL 1.1. Moreover, we provide reasoning support for a fuzzy version of fuzzy-$\mathcal{SROIQ}$ by extending well-known reduction techniques of fuzzy DLs to classical DLs for the additional axioms and constructors of fuzzy-$\mathcal{SROIQ}$.

1 Introduction

Although, OWL 1.1 [3] and Description Logics [1] are considerably expressive they are rather weak when it comes to modelling domains where imprecise and vague information is apparent. For that reasons there have been many proposals towards extending Description Logics and OWL DL with imprecise handling mathematical theories, resulting to fuzzy Description Logics [14] and fuzzy OWL [16, 11]. Let us consider for example the case of multimedia processing and analysis. Today a huge amount of multimedia documents, like image, video and sound records, reside in huge databases of TV channels, production companies, museums, galleries etc. In order to publish these archives on the web in a semantically rich manner we have to (semi)automatical annotate their content. In order to (semi)automatically annotate an image we have to employ an image analysis algorithm, which segments it in various regions (segments) and then associate with each segment a suitable semantic label, which will be for the purposes of retrieval. Unfortunately, the process of (semi)automatic image segmentation and recognition is an extremely difficult problem where a high degree of uncertainty and vagueness often appears. In order to assist image analysis the concept of

[(*)] This is a revised version of the paper presented in OWL ED 2007 knowledge based image analysis has been proposed [6]. More precisely, we can use expressive ontology languages, like $\mathcal{SROIQ}$ [4] or the respective OWL 1.1 [3], in order to give definitions about the entities that exist within an image. For example, we can have axioms like the following ones,

$$\begin{aligned}
\mathsf{Car} &\equiv \exists \mathsf{hasSegment}.(\mathsf{Body} \sqcap \exists \mathsf{isConnectedTo}.\mathsf{Wheel}) \\
\mathsf{Wheel} &\equiv \mathsf{Black} \sqcap \exists \mathsf{isConnectedTo}.\mathsf{WheelRim} \\
\mathsf{hasSegment} \circ \mathsf{isConnectedTo} &\sqsubseteq \mathsf{hasSegment}
\end{aligned}$$

Suppose now that we employ an image analysis algorithm. This algorithm segments the image and provides estimations on the membership or non-membership of a segment to a certain class. For example, by using the fuzzy DL syntax [14], we can have that $(region_1, region_2) : \mathsf{hasSegment} \geq 0.7$, $region_2 : \mathsf{Body} \geq 0.8$, $(region_2, region_3) : \mathsf{isConnectedTo} \geq 0.6$ and $region_3 : \mathsf{Wheel} \geq 0.9$. From this fuzzy knowledge we can, on one hand by using standard fuzzy-DL reasoning [14], deduce that $region_1 : \mathsf{Car} \geq 0.6$, while on the other hand by using the newly introduced complex role inclusion we can also infer that $region_1 : \exists \mathsf{hasSegment}.\mathsf{Wheel} \geq 0.6$.

In the current paper we extend several results presented in the literature about fuzzy extensions to Description Logics and OWL. More precisely, we extend the semantics of fuzzy-$\mathcal{SHOIN}$, presented in [16], to provide semantics for fuzzy-$\mathcal{SROIQ}$. Furthermore, in order to provide some initial support for reasoning in f_{KD}-$\mathcal{SROIQ}$ (see section 3 for a definition) we extend the mapping presented in [2] that reduces the satisfiability of f_{KD}-$\mathcal{SHOIN}$ to satisfiability of crisp $\mathcal{SHOIN}$ in order to cover the new features of fuzzy-$\mathcal{SROIQ}$. Moreover, we provide an overview of some recently developed reasoning systems for fuzzy DLs. Finally, we extend the abstract and XML syntax, semantics and reduction of fuzzy OWL DL to fuzzy-$\mathcal{SHOIN}$ presented in [11] to provide syntax and semantics of fuzzy OWL 1.1.

2 Fuzzy Set Preliminaries

Fuzzy set theory and fuzzy logic are widely used for capturing imprecise knowledge [5]. While in classical set theory an element either belongs to a set or not, in fuzzy set theory elements belong only to a certain degree. More formally, let X be a set of elements. A fuzzy subset A of X, is defined by a *membership function* $\mu_A(x)$, or simply $A(x)$ [5]. This function assigns any $x \in X$ to a value between 0 and 1 that represents the degree in which this element belongs to X. In this new framework the classical set theoretic and logical operations are performed by special mathematical functions. More precisely *fuzzy complement* is a unary operation of the form $c : [0,1] \to [0,1]$, *fuzzy intersection* and *union* are performed by two binary functions of the form $t : [0,1] \times [0,1] \to [0,1]$ and $u : [0,1] \times [0,1] \to [0,1]$, called *t-norm* and *t-conorm* operations [5], respectively, and *fuzzy implication* also by a binary function, $\mathcal{J} : [0,1] \times [0,1] \to [0,1]$. In order to produce meaningfull fuzzy complements, conjunctions, disjunctions and implications, these functions must satisfy certain mathematical properties. For example the operators must satisfy the following boundary properties, $c(0) = 1$, $c(1) = 0$, $t(1, a) = a$ and $u(0, a) = a$. Due to space limitations we cannot present all the properties that these functions should satisfy. The reader is referred to [5] for a comprehensive introduction. Examples of fuzzy operators are the Lukasiewicz negation, $c_L(a) = 1 - a$, t-norm, $t_L(a, b) = \max(0, a + b - 1)$, t-conorm $u_L(a, b) = \min(1, a + b)$, and implication, $\mathcal{J}_L(a, b) = \min(1, 1 - a + b)$, the Gödel norms $t_G(a, b) = \min(a, b)$, $u_G(a, b) = \max(a, b)$, and implication $\mathcal{J}_G(a, b) = b$ if $a > b$, 1 otherwise, and the Kleene-Dienes implication (KD-implication), $\mathcal{J}_{KD}(a, b) = \max(1 - a, b)$.

Finally, lets turn our attention to properties of fuzzy relations. A fuzzy relation R over $X \times X$ is called $\sup -t$ *transitive*, or simply *transitive* if $\forall a, b \in X, R(a, c) \geq \sup_{b \in X}\{t(R(a, b), R(b, c))\}$. R is *reflexive* if $\forall a \in X, R(a, a) = 1$, while it is called *irreflexive* if $\forall a \in X, R(a, a) = 0$.[1] In fuzzy set theory we are able to define a more weak notion of reflexivity, that of ϵ-reflexivity. Thus, R is ϵ-reflexive if $\forall a \in X, R(a, a) \geq \epsilon$. The *inverse* of a fuzzy relation $R : X \times Y \to [0, 1]$ is a fuzzy relation $R^- : Y \times X \to [0, 1]$ defined as $R^-(b, a) = R(a, b)$. Finally, given two fuzzy relations $R_1 : X \times Y \to [0, 1]$ and $R_2 : Y \times Z \to [0, 1]$ we define the $\sup -t$ composition as, $[R_1 \circ^t R_2](a, c) = \sup_{b \in Y}\{t(R(a, b), R(b, c))\}$. The operation of $\sup -t$ composition satisfies the following properties:

$$(R_1 \circ^t R_2) \circ^t R_3 = R_1 \circ^t (R_2 \circ^t R_3), \qquad (R_1 \circ^t R_2)^- = (R_2^- \circ^t R_1^-)$$

Due to the associativity property we can extend the operation of $\sup -t$ composition to any number of fuzzy relations. In that case we will simply write $[R_1 \circ^t R_2 \circ^t \ldots \circ^t R_n](a, b)$.

3 The Fuzzy $\mathcal{SROIQ}$ DL

In this section we introduce a fuzzy extension of the $\mathcal{SROIQ}$ DL, creating the fuzzy-$\mathcal{SROIQ}$ (f-$\mathcal{SROIQ}$) language. Due to space limitations and since fuzzy concrete domains have been introduced in [16] we will not present them here again. The reader is referred to [16] for more details. We are also using the notion of *fuzzy nominals* introduced in [2].

As usual we have an alphabet of distinct concept names ($\mathbf{C}$), role names ($\mathbf{R_A}$) (including the universal role U) and individuals ($\mathbf{I_A}$). The set of $\mathcal{SROIQ}$-*roles* is defined by $\mathbf{R_A} \cup \{R^- \mid R \in \mathbf{R_A}\}$, where R^- is called the *inverse role* of R. Let $A \in \mathbf{C}$, $R, S \in \mathbf{R_A}$ where S is a *simple* role [4], $o_i \in \mathbf{I_A}$, $n_i \in [0, 1]$ for $1 \leq i \leq m$ and $p \in \mathbb{N}$, then *f-$\mathcal{SROIQ}$-concepts* are defined inductively by the following production rule:

$$\begin{aligned} C, D \longrightarrow\ & \bot \mid \top \mid A \mid C \sqcup D \mid C \sqcap D \mid \neg C \mid \forall R.C \mid \exists R.C \mid \geq pS.C \mid \leq pS.C \mid \\ & \exists S.\mathsf{Self} \mid \{(o_1, n_1), \ldots, (o_m, n_m)\} \end{aligned}$$

[1] Note that in most fuzzy textbooks this property is referred to as antireflexivity, but in order to be aligned with OWL 1.1 axioms we call it irreflexivity.

A fuzzy *TBox* is a finite set of general concept inclusions (GCIs) of the form $C \sqsubseteq D$ between two f-$\mathcal{SROIQ}$-concepts C and D. Concept equivalence $C \equiv D$ can be captured by two inclusions $C \sqsubseteq D$ and $D \sqsubseteq C$. A fuzzy *ABox* is a finite set of fuzzy assertions. A *fuzzy assertion* [14] is of the form $(a : C) \bowtie n$, $((a, b) : R) \bowtie n$ where $\bowtie \in \{\geq, >, \leq, <\}$, $a = b$ or $a \neq b$, for $a, b \in \mathbf{I_A}$. We use $\bowtie^-$ as the *reflection* of inequalities, e.g. $\geq^- = \leq$ and $<^- = >$.

Differently than crisp $\mathcal{SROIQ}$, we have not explicitly defined simple negation on roles. That is because this kind of expressivity implicitly exists in fuzzy DL systems by mean of assertions that use the inequalities, $\leq$ and $<$ [14]. More precisely a statement of the form "John does not like Mary" can be defined by the assertion, $((\mathsf{John}, \mathsf{Mary}) : \mathsf{likes}) \leq 0$. Such assertions are being handled by fuzzy DL reasoners [13].

A fuzzy *RBox* consists of two components. The first one is a *role hierarchy* $\mathcal{R}_h$, which consists of (generalized) *role inclusion axioms* and the second one is a set $\mathcal{R}_a$ of *role assertions* [4]. A role inclusion axiom (RIA) is an axiom of the form $R_1 \dots R_n \sqsubseteq S$, where $R_1, \dots, R_n, S$ are f-$\mathcal{SROIQ}$-roles. Intuitively, such axioms state that the composition of roles $R_1, \dots, R_n$ imply the role S. For roles $R, S \neq U$, the role axioms, $\mathsf{Trans}(R)$, $\mathsf{Ref}(R)$, ϵ-$\mathsf{Ref}(R, n)$, $\mathsf{Irr}(R)$, $\mathsf{Sym}(R)$, $\mathsf{ASym}(R)$, and $\mathsf{Dis}(R, S)$ are called *role assertions* [4]. Intuitively, these axioms state that R is transitive, reflexive, ϵ-reflexive, irreflexive, symmetric, antisymmetric, and disjoint from S, respectively. Compared to $\mathcal{SROIQ}$ role assertions [4] ϵ-reflexivity is obviously a new role assertion. A fuzzy knowledge base Σ is a triple $\langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$, that contains a fuzzy *TBox*, *RBox* and *ABox*, respectively.

The semantics of fuzzy DLs are provided by a *fuzzy interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ [14], where the domain $\Delta^{\mathcal{I}}$ is a non-empty set of objects and $\cdot^{\mathcal{I}}$ is a fuzzy interpretation function, which maps: (i) an individual a to an element $\mathsf{a}^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, (ii) a concept name A to a function $\mathsf{A}^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0, 1]$, and (iii) a role name R to a function $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$,

Definition 1 (Concept Descriptions, TBox, RBox, ABox). *Given an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, concepts $C, D \in \mathbf{C}$, roles $R, S \in \mathbf{R_A}$, objects $a, b \in \Delta^{\mathcal{I}}$, $n_i \in [0, 1]$, for $1 \leq i \leq m$, and $p \in \mathbb{N}$ the interpretation of complex f-$\mathcal{SROIQ}$-concepts is defined inductively by the following equations:*

$$\begin{aligned}
\bot^{\mathcal{I}}(a) &= 0, & \top^{\mathcal{I}}(a) &= 1, \\
(C \sqcap D)^{\mathcal{I}}(a) &= t(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)), & (C \sqcup D)^{\mathcal{I}}(a) &= u(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)), \\
(\neg C)^{\mathcal{I}}(a) &= c(C^{\mathcal{I}}(a)), & \{(o_i, n_i)\}^{\mathcal{I}}(a) &= \sup_{i \mid a \in \{o_i^{\mathcal{I}}\}} n_i,\ 1 \leq i \leq m, \\
(\exists R.C)^{\mathcal{I}}(a) &= \sup\nolimits_{b \in \Delta^{\mathcal{I}}} t(R^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b)), & (\exists R.\mathsf{Self})^{\mathcal{I}}(a) &= R^{\mathcal{I}}(a, a), \\
(\forall R.C)^{\mathcal{I}}(a) &= \inf\nolimits_{b \in \Delta^{\mathcal{I}}} \mathcal{J}(R^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b)), & &
\end{aligned}$$

$$(\geq pR.C)^{\mathcal{I}}(a) = \sup_{b_1, \dots, b_p \in \Delta^{\mathcal{I}}} t(\underset{i=1}{\overset{p}{t}} \{t(R^{\mathcal{I}}(a, b_i), C^{\mathcal{I}}(b_i))\}, \underset{i<j}{t} \{b_i \neq b_j\}),$$

$$(\leq pR.C)^{\mathcal{I}}(a) = \inf_{b_1, \dots, b_{p+1} \in \Delta^{\mathcal{I}}} \mathcal{J}(\underset{i=1}{\overset{p+1}{t}} \{t(R^{\mathcal{I}}(a, b_i), C^{\mathcal{I}}(b_i))\}, \underset{i<j}{u} \{b_i = b_j\}),$$

Additionally, the fuzzy interpretation function assigns the universal role U the membership function $U^{\mathcal{I}}(a, b) = 1$, for each $\langle a, b \rangle \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

An interpretation $\mathcal{I}$ satisfies a GCI $C \sqsubseteq D$, written $\mathcal{I} \models C \sqsubseteq D$, if $\forall a \in \Delta^{\mathcal{I}}.C^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a)$. If satisfies each GCI in $\mathcal{T}$ then we say that $\mathcal{I}$ is a model *of $\mathcal{T}$.*

Furthermore, for each fuzzy interpretation $\mathcal{I}$ and all $a, b, c \in \Delta^{\mathcal{I}}$ we have,

$$
\begin{array}{lll}
\mathcal{I} \models \mathsf{Trans}(R) & \textit{if} & R^{\mathcal{I}}(a,c) \geq sup_{b\in\Delta^{\mathcal{I}}}\{t(R^{\mathcal{I}}(a,b), R^{\mathcal{I}}(b,c))\}, \\
\mathcal{I} \models \mathsf{Ref}(R) & \textit{if} & R^{\mathcal{I}}(a,a) = 1, \\
\mathcal{I} \models \epsilon\text{-}\mathsf{Ref}(R,n) & \textit{if} & R^{\mathcal{I}}(a,a) \geq n, \\
\mathcal{I} \models \mathsf{Irr}(R) & \textit{if} & R^{\mathcal{I}}(a,a) = 0, \\
\mathcal{I} \models \mathsf{Sym}(R) & \textit{if} & R^{\mathcal{I}}(a,b) = R^{\mathcal{I}}(b,a), \\
\mathcal{I} \models \mathsf{ASym}(R) & \textit{if} & R^{\mathcal{I}}(a,b) \neq R^{\mathcal{I}}(b,a) \\
\mathcal{I} \models \mathsf{Dis}(R,S) & \textit{if} & t(R^{\mathcal{I}}(a,b), S^{\mathcal{I}}(a,b)) = 0, \\
\mathcal{I} \models R_1 \ldots R_n \sqsubseteq S & \textit{if} & [R_1^{\mathcal{I}} \circ^t \ldots \circ^t R_n^{\mathcal{I}}](a,b) \leq S^{\mathcal{I}}(a,b),
\end{array}
$$

Additionally, an inverse role R^- of R is interpreted as $(R^-)^{\mathcal{I}}(a,b) = R^{\mathcal{I}}(b,a)$. In case where $\mathcal{I}$ satisfies each axiom in $\mathcal{R}$ we say that $\mathcal{I}$ is a model of $\mathcal{R}$.

Finally, $\mathcal{I}$ satisfies $(a : C) \geq n$ and $((a,b) : R) \geq n$ if $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ and $R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \geq n$, while $\mathcal{I}$ satisfies $a = b$ if $a^{\mathcal{I}} = b^{\mathcal{I}}$ and it satisfies $a \neq b$ if $a^{\mathcal{I}} \neq b^{\mathcal{I}}$. The satisfiability of fuzzy assertions with $\leq, >$ and $<$ is defined analogously. A fuzzy interpretation satisfies a fuzzy ABox $\mathcal{A}$ if it satisfies all fuzzy assertions in $\mathcal{A}$. In this case, we say $\mathcal{I}$ is a model *of $\mathcal{A}$. Finally, a fuzzy interpretation $\mathcal{I}$ satisfies an f-$\mathcal{SROIQ}$ knowledge base Σ if it satisfies all axioms in Σ; in this case, $\mathcal{I}$ is called a* model *of Σ.*

As we can see RIAs are interpreted as the $\sup -t$ compositions of fuzzy relations. Hence, from the properties of the $\sup -t$ composition and the semantics of inverse roles it holds that if $\mathcal{I}$ satisfies $R_1 \ldots R_n \sqsubseteq S$, then it also satisfies $\mathsf{Inv}(R_n) \ldots \mathsf{Inv}(R_1) \sqsubseteq \mathsf{Inv}(S)$. Thus the semantics in the fuzzy case are aligned with the crisp semantics of RIAs.

As it is shown in [4], $\mathcal{SROIQ}$ has much expressive power to encode role assertions $\mathsf{Irr}(R)$, $\mathsf{Ref}(R)$, $\mathsf{Trans}(R)$, or $\mathsf{Sym}(R)$ with the aid of RIAs or by using the new special concept $\exists R.\mathsf{Self}$. We can prove that the same situation holds in the case of fuzzy $\mathcal{SROIQ}$. Finally, it is also worth noting that the axioms of ϵ-reflexivity (ϵ-$\mathsf{Ref}(R,n)$) cannot be eliminated.

Concluding this presentation we introduce some notation. As it is evident different choices of fuzzy operators define different fuzzy DL languages. For that reason a special notation is needed in order to distinguish between such different f-DL languages. In [10] the notation $\mathrm{f}_{\mathcal{J}}$-$\mathcal{L}$ is used, where $\mathcal{J}$ is a fuzzy implication and $\mathcal{L}$ is a DL language. So for example, f_{KD}-$\mathcal{SROIQ}$, is the fuzzy $\mathcal{SROIQ}$ language which uses the Kleene-Dienes fuzzy implication ($\mathcal{J}(a,b) = \max(1-a,b)$), while the rest of the operators are the defined ones, i.e. the Gödel t-conorm ($u(a,b) = \max(a,b)$), the Lukasiewicz negation ($c(a) = 1-a$) Gödel t-norm $t(a,b) = c(u(c(a),c(b))) = \min(a,b)$. Similarly, f_L-$\mathcal{SROIQ}$ is f-$\mathcal{SROIQ}$ which uses the Lukasiewicz implication, t-norm, t-conorm and negation.

4 Reasoning in fuzzy DLs

One of the main concerns for applying fuzzy DLs in applications was the lack of fuzzy DL reasoning systems and algorithms. Fortunately, lately there is a growing interest and effort in the field which leaded to the creation of many interesting reasoning platforms. In the current section we will review some recently developed reasoning systems for fuzzy DLs and finally, we will extend a reasoning technique proposed for fuzzy DLs to the case of f-$\mathcal{SROIQ}$.

There are many different proposals to perform reasoning in fuzzy DLs. Stoilos et. al. [10, 9] develop direct tableaux methods for reasoning in very expressive f-DLs, like the f$_{KD}$-$\mathcal{SI}$ and f$_{KD}$-$\mathcal{SHIN}$, respectively. Some first ideas for reasoning in f$_{KD}$-$\mathcal{SHOIN}$ are also presented in [11]. The f$_{KD}$-$\mathcal{SHIN}$ algorithm has been implemented in the FiRE platform [13] which is available for testing at `http://www.image.ece.ntua.gr/~nsimou`. We have to mention that currently the implementation works only on simple TBoxes (no GCIs or cycles are allowed), while the extension to allow for GCIs and cycles is investigated after the new results obtained for them [12]. On the other hand Straccia uses an optimization technique, called mixed integer linear programming, to provide reasoning for the f$_{KD}$-$\mathcal{ALC}(\mathcal{D})$ and f$_L$-$\mathcal{ALC}(\mathcal{D})$ languages [15]. The optimization technique seems a right choice to generalize to other norm operators, like the f$_L$-DLs, since differently than f$_{KD}$-DLs, but reasoning involves external calls to equation solvers. Recently the ideas in [15] have been applied to $\mathcal{SHIF}(\mathcal{D})$ to provide a reasoner for f$_L$-$\mathcal{SHIF}(\mathcal{D})$ and f$_{KD}$-$\mathcal{SHIF}(\mathcal{D})$ (available at `http://gaia.isti.cnr.it/~straccia`) but the theoretical details of the implementation are not yet available. Finally, Straccia proposed in [17] an additional way to perform reasoning in f$_{KD}$-DLs. This technique actually reduces an f$_{KD}$-$\mathcal{L}$ knowledge base to a crisp $\mathcal{L}$ KB and uses well-known classical DL systems to provide indirectly reasoning support for f$_{KD}$-DLs. Straccia shown the case of f$_{KD}$-$\mathcal{ALCH}$, while the technique has been recently generalized to cover the f$_{KD}$-$\mathcal{SHOIN}$ DL [2]. Here we will use the reduction technique to provide reasoning support for f$_{KD}$-$\mathcal{SROIQ}$.

The main idea behind the reduction technique is that a fuzzy assertion of the form $(a : C) \geq n$, where a is an individual and $n \in [0, 1]$ can be represented by a crisp assertion of the form $a : C_{\geq n}$, where $C_{\geq n}$ is a new crisp concept. In order for the reduction to be satisfiability preserving we also have to capture the semantic relation between two concepts of the form $C_{\geq n_1}$ and $C_{\geq n_2}$. For example, if $n_1 \leq n_2$ it is obvious that $C_{\geq n_2} \sqsubseteq C_{\geq n_1}$, while for each degree n_1 it should hold that $C_{\geq n_1} \sqcap C_{<n_1} \sqsubseteq \bot$, $C_{>n_1} \sqcap C_{\leq n_1} \sqsubseteq \bot$, $\top \sqsubseteq C_{>n_1} \sqcup C_{\leq n_1}$ and $\top \sqsubseteq C_{\geq n_1} \sqcup C_{<n_1}$. Similarly we have to work with roles. In the following we will provide the necessary extensions to the reduction in [2] in order to be able to translate f$_{KD}$-$\mathcal{SROIQ}$ knowledge bases to crisp $\mathcal{SROIQ}$ knowledge bases.

Let $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$ be a fuzzy knowledge base. Then, we define $N^{\Sigma} = \{0, 0.5, 1\} \cup \{n, 1 - n \mid (a : C) \bowtie n \text{ or } ((a, b) : R) \bowtie n\}$ [17]. The reason why we can restrict our attention to only these specific degrees is that in f$_{KD}$-DLs if there exists a model for a fuzzy knowledge base, then there is also a model using only these degrees.

Let $\mathcal{R}$ be an f_{KD}-$\mathcal{SROIQ}$ RBox. The function κ from [2] is extended from transitive role axioms to the additional axioms of $\mathcal{SROIQ}$, while the function ρ is extended to the new special concept of the form $\exists R.\mathsf{Self}$ in the following way:

$$\begin{aligned}
\kappa(\mathsf{Ref}(R)) &= \mathsf{Ref}(R_{\geq 1}),\\
\kappa(\epsilon\text{-}\mathsf{Ref}(R,n)) &= \mathsf{Ref}(R_{\geq n}),\\
\kappa(\mathsf{Irr}(R)) &= \bigcup_{n\in N^{\Sigma}} \mathsf{Irr}(R_{>n}) \cup \bigcup_{c\in N^{\Sigma}\setminus\{0\}} \mathsf{Irr}(R_{\geq n}),\\
\kappa(\mathsf{Sym}(R)) &= \bigcup_{n\in N^{\Sigma},\bowtie\in\{\geq,>\}} \mathsf{Sym}(R_{\bowtie n}),\\
\kappa(\mathsf{ASym}(R)) &= \bigcup_{n\in N^{\Sigma},\bowtie\in\{\geq,>\}} \mathsf{ASym}(R_{\bowtie n}),\\
\kappa(\mathsf{Dis}(R,S)) &= \mathsf{Dis}(R_{>0},S_{>0}),\\
\kappa(R_1\ldots R_m \sqsubseteq S) &= \bigcup_{n\in N^{\Sigma},\bowtie\in\{\geq,>\}} R_{1_{\bowtie n}}\ldots R_{m_{\bowtie n}} \sqsubseteq S_{\bowtie n}\\
\rho(\exists R.\mathsf{Self},\bowtie n) &= \exists R_{\bowtie n}.\mathsf{Self} \text{ if } \bowtie = \{\geq,>\}
\end{aligned}$$

It is very important to point out that the above reduction, as well as the ones in [17, 2] only hold for f_{KD}-DLs.

Concluding our presentation of fuzzy DL reasoning algorithms and systems, it is worth noting that following the trend of tractable fragments of DLs, fuzzy DLs with polynomial complexity have also been investigated. More precisely, Straccia proves in [18] that f_{KD}-DL-Lite is still polynomial and the reasoning technique is very similar to the one of crisp DL-Lite with few modifications in the procedures of ABox storing and KB consistency. In [7] we have implemented the f_{KD}-DL-Lite algorithm in the ONTOSEARCH2 platform, while we have also extended the query language from [18] to include many new expressive features like preferences and thresholds in query atoms.

5 Fuzzy OWL 1.1

In [11] the abstract syntax for fuzzy individual axioms (fuzzy facts) of fuzzy OWL DL was presented. Here we extend this abstract syntax to also include the new features of OWL 1.1 like the simple negation on roles, while we also extend the definition of enumerated classes, that was not provided in [11] and [2] to represent fuzzy nominals in OWL. The extended definition is presented in Table 2, where we have abbreviated some very long names.

Based on the above extensions we can serialize the extended abstract syntax to provide an XML syntax for fuzzy OWL 1.1. More precisely, we use the elements `owlx:ineqType` and `owlx:degree` [11, 13] for providing the inequality type and the membership degree. Then, we can encode fuzzy facts, like the ones about image segments, as

```
<Body rdf:about="region2" owlx:ineqType="≥" owlx:degree="0.8">
  <isConnectedTo rdf:resource="region3" owlx:degree="0.6"/>
</HotPlace>
```

Table 1. New Fuzzy OWL 1.1 Class Descriptions and Axioms

Abstract Syntax	DL Syntax	Semantics
ObjectOneOf($(o_1, n_1), (o_2, n_2), \ldots$)	$\{(o_1, n_1), (o_2, n_2)\}$	$\{(o_1, n_1), (o_2, n_2)\}^{\mathcal{I}}(a) = \sup_{i \mid a \in \{o_i^{\mathcal{I}}\}} n_i$
ObjectHasValue(R o))	$\exists R.\{$(o,1)$\}$	$(\exists R.\{(o, 1)\})^{\mathcal{I}}(a) = \sup_{b \in \Delta^{\mathcal{I}}} t(R^{\mathcal{I}}(a, b), \{(o, 1)\}^{\mathcal{I}}(b))$
ObjectExistsSelf(R)	$\exists R.\mathsf{Self}$	$(\exists R.\mathsf{Self})^{\mathcal{I}}(a) = R^{\mathcal{I}}(a, a)$
ObjectMinCardinality(R p C))	$\geq pR.C$	$(\geq pR.C)^{\mathcal{I}}(a) = \sup_{b_1, \ldots, b_p \in \Delta^{\mathcal{I}}} t(\mathop{\mathrm{t}}_{i=1}^{p} \{t(R^{\mathcal{I}}(a, b_i), C^{\mathcal{I}}(b_i))\}, \mathop{\mathrm{t}}_{i<j} \{b_i \neq b_j\})$
ObjectMaxCardinality(R p C))	$\leq pR.C$	$(\leq pR.C)^{\mathcal{I}}(a) = \inf_{b_1, \ldots, b_{p+1} \in \Delta^{\mathcal{I}}} \mathcal{J}(\mathop{\mathrm{t}}_{i=1}^{p+1} \{t(R^{\mathcal{I}}(a, b_i), C^{\mathcal{I}}(b_i))\}, \mathop{\mathrm{u}}_{i<j} \{b_i = b_j\})$
ObjectExactCardinality(R p C))	$\geq pR.C \sqcap \leq pR.C$	$(\geq pR.C \sqcap \leq pR.C)^{\mathcal{I}}(a) = t((\geq pR.C)^{\mathcal{I}}(a), (\leq pR.C)^{\mathcal{I}}(a))$
DisjointUnion(C $C_1 \ldots C_n$)	$C \equiv C_1 \sqcup \ldots \sqcup D_n$,	$C^{\mathcal{I}}(a) = u(C_1^{\mathcal{I}}(a), \ldots, C_2^{\mathcal{I}}(a))$,
	$C_i \sqcap C_j \sqsubseteq \bot$	$t(C_1^{\mathcal{I}}(a), C_j^{\mathcal{I}}(a)) = 0 \; 1 \leq i < j \leq n$
ReflexiveObjectProperty(R)	$\mathsf{Ref}(R)$	$R^{\mathcal{I}}(a, a) = 1$
IrreflexiveObjectProperty(R)	$\mathsf{Irr}(R)$	$R^{\mathcal{I}}(a, a) = 0$
AntisymmetricObjectProperty(R)	$\mathsf{ASym}(R)$	$t(R^{\mathcal{I}}(a, b), R^{\mathcal{I}}(b, a)) = 0$
SubObjectPropertyOf(SubObjectPropertyChain($R_1 \ldots R_n$) S)	$R_1 \ldots R_n \sqsubseteq S$	$R_1^{\mathcal{I}}(a, y_1) \circ^t \ldots \circ^t R_n^{\mathcal{I}}(y_n, b) \leq S^{\mathcal{I}}(a, b)$
DisjointObjectProperties(R_1 ... R_n)	$\mathsf{Dis}(R_i, R_j)$	$t(R_i^{\mathcal{I}}(a, b), R_j^{\mathcal{I}}(a, b)) = 0,\ 1 \leq i < j \leq n$
ClassAssertion(o type(C) $\bowtie$ degree(n))	$(o : C) \bowtie n$	$C^{\mathcal{I}}(o^{\mathcal{I}}) \bowtie n, n \in [0, 1]$
ObjectPropertyAssertion(R o_1 o_2 $\bowtie$ degree(n))	$((o_1, o_2) : R) \bowtie n$	$R^{\mathcal{I}}(o_1^{\mathcal{I}}, o_2^{\mathcal{I}}) \bowtie n, n \in [0, 1]$
NegativeObjectPropertyAssertion(R o_1 o_2 $\bowtie$ degree(n))	$((o_1, o_2) : R) \bowtie^{-} c(n)$	$R^{\mathcal{I}}(o_1^{\mathcal{I}}, o_2^{\mathcal{I}}) \bowtie^{-} c(n), n \in [0, 1]$
Sameindividual($o_1 \ldots o_n$)	$o_1 = \ldots = o_n$	$o_1^{\mathcal{I}} = \ldots = o_n^{\mathcal{I}}$
DifferentIndividuals($o_1 \ldots o_n$)	$o_i \neq o_j$	$o_i^{\mathcal{I}} \neq o_j^{\mathcal{I}}, 1 \leq i < j \leq n$

or define classes with enumeration of fuzzy nominals like the German speaking countries as,

```
<owl:Class rdf:ID="GermanSpeaking">
  <owl:oneOf rdf:parseType="Collection">
    <Country rdf:about="#Germany" owlx:degree="1"/>
    <Country rdf:about="#Austria" owlx:degree="1"/>
    <Country rdf:about="#Switzerland" owlx:degree="0.67"/>
  </owl:oneOf>
```

Table 2. Abstract Syntax of f-OWL 1.1

```
classAssertion  ::= 'ClassAssertion(' { annotation } individualURI description membership ')'
objPropAss      ::= 'ObjectPropertyAssertion(' { annotation } objectPropExpression
                    sourceIndividualURI targetIndividualURI membership ')'
negObjPropAss   ::= 'NegativeObjectPropertyAssertion(' { annotation } objectPropExp
                    sourceIndividualURI targetIndividualURI membership')'
objectOneOf     ::= 'ObjectOneOf(' individualURI [degree] { individualURI [degree]} ')'
membership      ::= [ineqType] [degree]
ineqType        ::= '=' | '>=' | '>' | '<=' | '<'
degree          ::= real-number-between-0-and-1-inclusive
```

```
</owl:Class>
```

Furthermore the direct model-theoretic semantics of f-OWL 1.1 are provided by extensions of interpretations, i.e. fuzzy interpretation, which are similar to the ones introduced in section 3. An f-OWL interpretation can be extended to give semantics to fuzzy concept and object property descriptions and axioms. The complete set of semantics is depicted in Table 1, where a, b are arbitrary objects of $\Delta^{\mathcal{I}}$. As we can see, although we use fuzzy nominals in enumerated classes we do not allow them in hasValue restrictions. This constructor originates from the *fills* constructor, whose DL syntax is $R : o$ and semantics $(R : o)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid (d, o^{\mathcal{I}}) \in R^{\mathcal{I}}\}$ [1]. Intuitively, an assertion $a : (R : o)$ intends to capture that a is connected with a specific individual (o) through R. This constructor is a syntactic sugar in the presence of nominals and existential restrictions in the crisp case, written as $\exists R.\{o\}$. A natural way to give semantics to the fills constructor in the fuzzy case is through the equation $(R : o)^{\mathcal{I}}(d) = R^{\mathcal{I}}(d, o^{\mathcal{I}})$, which is different than the semantics of $a : \exists R.\{(o, n)\}$. Still the extension is trivial. Moreover note that since we have not defined simple negation on roles a fuzzy facts of the form negativeObjectPropertyAssertion($R\ a\ b \geq n$), is translated to the fuzzy assertions $((a, b) : R) \geq^{-} c(n)$, i.e. $((a, b) : R) \leq c(n)$.

6 Conclusions

In the current paper we present a fuzzy extension to the $\mathcal{SROIQ}$ DL and the OWL 1.1 Semantic Web language. We believe that such extensions are very important since on the one hand there are many applications where information is inherently imprecise and vague, hence these extensions would make such technologies more easily adoptable by applications that have not yet, but want, to enter the Semantic Web era. On the other hand fuzzy extensions might also be of interest to the researchers of the Semantic Web since they can be used in order to model problems where information is vague and various types of degrees appears, like querying with preferences or querying distributed information sources which are assigned different degrees of trust or confidence.

Regarding future work, we are planning to extend the reasoning algorithm of f_{KD}-$\mathcal{SHIN}$ [9] and f_{KD}-$\mathcal{SHOIN}$ [11], to develop a tableaux decision procedure that will provide direct reasoning support (compared to the reduction) for f_{KD}-$\mathcal{SROIQ}$. Moreover, the issue of reasoning with qualified cardinality restriction ($\mathcal{Q}$) based on the semantics for number restrictions proposed in [16] (see also [8] for the qualified case) is still open.

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