Reasoning with the Fuzzy Description Logic f-$\mathcal{SHIN}$: Theory, Practice and Applications

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Abstract. The last couple of years it is widely acknowledged that uncertainty and fuzzy extensions to ontology languages, like Description Logics (DLs) and OWL, could play a significant role in the improvement of many Semantic Web (SW) applications. Many of the tasks of SW like trust, matching, merging, ranking usually involve confidence or truth degrees that one requires to represent and reason about. Fuzzy DLs are able to represent vague concepts such as a "Tall" person, a "Hot" place, a "MiddleAged" person, a "near" destination and many more. In the current paper we present a fuzzy extension to the DL $\mathcal{SHIN}$. First, we present the semantics while latter a detailed reasoning algorithm that decides most of the key inference tasks of fuzzy-$\mathcal{SHIN}$. Finally, we briefly present the fuzzy reasoning system FiRE, which implements the proposed algorithm and two use case scenarios where we have applied fuzzy DLs through FiRE.

1 Introduction

The last decade a significant amount of research has been focused in the development of the Semantic Web [1]. Semantic Web actually consists of an extension of the current Web where information, that lies in databases, web pages, etc., would be semantically accessible, enabling complex tasks to be performed in an (semi)automatic way. For example, Semantic Web agents would be able to accomplish tasks like a holiday organization, a doctor appointment, the retrieval of images depicting specific events etc. in a semantic and (semi)automatic way. In order to accomplish this goal it is widely recognized that information on the web should be structured in a machine understandable way, by using knowledge representation languages and forming *ontologies* [1]. For those reasons W3C[1] has standardized a number of ontology (knowledge representation) languages for the Web. One of the most important and expressive ones is OWL [2]. The logical underpinnings of OWL consist of very expressive Description Logics [3] and more precisely, the OWL DL species of OWL is equivalent to $\mathcal{SHOIN}(\mathbf{D}^+)$, while OWL Lite is equivalent to $\mathcal{SHIF}(\mathbf{D}^+)$.

[1] `http://www.w3.org/`

P.C.G. da Costa et al. (Eds.): URSW 2005-2007, LNAI 5327, pp. 262–281, 2008.

Although, Description Logics are relatively expressive, they are based on two-valued (Boolean) logics, which consider everything either true or false, thus they are unable to represent truth degrees, which are important in representing vague (fuzzy) knowledge. For example, they are unable to correctly represent concepts like a “tall” man, a “fast” car, a “blue” sky and many more. Moreover, many Semantic Web applications, like knowledge based information retrieval and ontology matching [4], also involve degrees of equivalence or similarity, which are important to represent and reason about. For those reasons fuzzy Description Logics [5, 6] and fuzzy OWL [7] have been proposed as languages capable of representing and reasoning with vague knowledge in the Semantic Web. With fuzzy DLs one usually is able to provide the same schema information as classical (crisp) DLs. For example, one can still define the concept of a MiddleAged person as someone that is either in his/her Forties or Fifties with the following axiom:

$$\mathsf{MiddleAged} \equiv \mathsf{Forties} \sqcup \mathsf{Fifties}$$

On the other hand, one is able to state that John is in his fifties to a degree at least 0.6 (since he is 46 years old) by writing $(john : \mathsf{Fifties}) \geq 0.6$, while he is also tall to a degree at least 0.8 (since he is 190cm), writing $(john : \mathsf{Tall}) \geq 0.8$. Apart from representing fuzzy knowledge one should be able to also reason about, and for example infer that John is middle aged to a degree at least 0.8.

In the current paper we report on some recent results obtained about reasoning in very expressive fuzzy Description Logics and more precisely about reasoning with the fuzzy DL f_{KD}-$\mathcal{SHIN}$ [6,8,9,10]. First, we present a tableaux reasoning algorithm for f_{KD}-$\mathcal{SHIN}$. Then, we report on an implementation of the algorithm which gave rise to the FiRE fuzzy DL system and consists of an extension of the tool presented in [8]. FiRE provides a graphical user interface that can be used to load and reason with fuzzy DL ontologies. Furthermore, FiRE is able to store a fuzzy knowledge into a triple store and query about it using very expressive fuzzy conjunctive queries [11]. The rest of the paper is organized as follows. Section 2 presents the syntax and semantics of the fuzzy extension of $\mathcal{SHIN}$. Then, in Section 3 we provide all the technical details for a reasoning algorithm that decides most of the inference problems of fuzzy-$\mathcal{SHIN}$. Subsequently, Section 4 provides a brief presentation of the FiRE system. Then, Section 5 presents two Use Case scenarios where we have applied FiRE and we discuss its potentials and future directions. Finally, Section 6 provides a discussion about state-of-the-art work in fuzzy DLs, while it also presents a list of important open problems related to the area of fuzzy Description Logics.

2 Syntax and Semantics of f-$\mathcal{SHIN}$

In this section we introduce the DL f-$\mathcal{SHIN}$. As usual we have an alphabet of distinct concept names ($\mathbf{C}$), role names ($\mathbf{R}$) and individual names ($\mathbf{I}$). f-$\mathcal{SHIN}$-roles and f-$\mathcal{SHIN}$-concepts are defined as follows:

Definition 1. *Let $RN \in \mathbf{R}$ be a role name, R an f-$\mathcal{SHIN}$-role. f-$\mathcal{SHIN}$-roles are defined by the abstract syntax: $R ::= RN \mid R^-$. The inverse relation of roles is symmetric, and to avoid considering roles such as R^{--}, we define a function* Inv*, which returns the inverse of a role, more precisely $\mathsf{Inv}(R) := RN^-$ if $R = RN$ and $\mathsf{Inv}(R) := RN$ if $R = RN^-$.*

The set of f-$\mathcal{SHIN}$-concepts is the smallest set such that:

1. *every concept name $CN \in \mathbf{C}$ is an f-$\mathcal{SHIN}$-concept,*
2. *if C and D are f-$\mathcal{SHIN}$-concepts and R is an f-$\mathcal{SHIN}$-role, then $(C \sqcup D)$, $(C \sqcap D)$, $(\neg C)$, $(\forall R.C)$ and $(\exists R.C)$ are also f-$\mathcal{SHIN}$-concepts,*
3. *if R is a* simple[2] *f-$\mathcal{SHIN}$-role and $p \in \mathbb{N}$, then $(\geq pR)$ and $(\leq pR)$ are also f-$\mathcal{SHIN}$-concepts.*

Although the definition of $\mathcal{SHIN}$-concepts and roles is the same with the one of fuzzy-$\mathcal{SHIN}$-concepts and roles the semantics of f-$\mathcal{SHIN}$ are significantly extended. This is because semantically we have to provide a fuzzy meaning/interpretation to the building blocks of our language, like concepts, roles and constructors. For that reason the semantics of fuzzy DLs are defined with the help of *fuzzy interpretations* [5]. A fuzzy interpretation is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where the domain $\Delta^{\mathcal{I}}$ is a non-empty set of objects and $\cdot^{\mathcal{I}}$ is a *fuzzy interpretation function*, which maps:

1. an individual name $a \in \mathbf{I}$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$,
2. a concept name $\mathsf{A} \in \mathbf{C}$ to a membership function $\mathsf{A}^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1]$,
3. a role name $RN \in \mathbf{R}$ to a membership function $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$.

Intuitively, an object (pair of objects) can now belong to a fuzzy concept (role) to any degree between 0 and 1. For example, $\mathsf{HotPlace}^{\mathcal{I}}(\mathtt{Athens}^{\mathcal{I}}) = 0.7$, means that $\mathtt{Athens}^{\mathcal{I}}$ is a hot place to a degree equal to 0.7. Additionally, a fuzzy interpretation function can be extended in order to provide semantics to any complex f-$\mathcal{SHIN}$-concept and role by using the operators of fuzzy set theory. More precisely, in the current setting we use the Lukasiewicz negation ($c(a) = 1-a$), Gödel conjunction ($\min(a,b)$) and disjunction ($\max(a,b)$) and the Kleene-Dienes fuzzy implication ($\max(1-a,b)$). Then, since $C \sqcup D$ represents a disjunction (union) between concepts C and D we can use max and provide the semantic function for disjunction: $(C \sqcup D)^{\mathcal{I}}(a) = u(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))$. The complete set of semantics for f-$\mathcal{SHIN}$-concepts and roles is depicted in Table 1. We remark that due to the operators we use we call our language f_{KD}-$\mathcal{SHIN}$.

An f_{KD}-$\mathcal{SHIN}$ *TBox* $\mathcal{T}$ is a finite set of *terminological axioms*. Let C and D be two f_{KD}-$\mathcal{SHIN}$-concepts. Axioms of the form $C \sqsubseteq D$ are called *fuzzy concept inclusion axioms* or *fuzzy concept subsumptions* or simply subsumptions, while axioms of the form $C \equiv D$ are called *fuzzy concept equivalence axioms*. A fuzzy interpretation $\mathcal{I}$ satisfies an axiom $C \sqsubseteq D$ if $\forall a \in \Delta^{\mathcal{I}}$, $C^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a)$ while it satisfies an axiom $C \equiv D$ if $C^{\mathcal{I}}(a) = D^{\mathcal{I}}(a)$. Finally, a fuzzy interpretation $\mathcal{I}$

[2] A role is called *simple* if it is neither transitive nor has any transitive sub-roles.

Table 1. Semantics of f$_{KD}$-$\mathcal{SHIN}$-concepts and f$_{KD}$-$\mathcal{SHIN}$-roles

Constructor	**Syntax**	**Semantics**
top	$\top$	$\top^{\mathcal{I}}(a) = 1$
bottom	$\bot$	$\bot^{\mathcal{I}}(a) = 0$
general negation	$\neg C$	$(\neg C)^{\mathcal{I}}(a) = 1 - C^{\mathcal{I}}(a)$
conjunction	$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}}(a) = \min(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))$
disjunction	$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}}(a) = \max(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))$
exists restriction	$\exists R.C$	$(\exists R.C)^{\mathcal{I}}(a) = \sup_{b \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))\}$
value restriction	$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(a) = \inf_{b \in \Delta^{\mathcal{I}}}\{\max(1 - R^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))\}$
at-most restriction	$\le pR$	$(\le pR)^{\mathcal{I}}(a) = \inf\limits_{b_1,\ldots,b_{p+1} \in \Delta^{\mathcal{I}}} \max\limits_{i=1}^{p+1}\{1 - R^{\mathcal{I}}(a,b_i)\}$
at-least restriction	$\ge pR$	$(\ge pR)^{\mathcal{I}}(a) = \sup\limits_{b_1,\ldots,b_p \in \Delta^{\mathcal{I}}} \min\limits_{i=1}^{p}\{R^{\mathcal{I}}(a,b_i)\}$
inverse roles	R^-	$(R^-)^{\mathcal{I}}(b,a) = R^{\mathcal{I}}(a,b)$

satisfies an f$_{KD}$-$\mathcal{SHIN}$ TBox $\mathcal{T}$ if it satisfies every axiom in $\mathcal{T}$. Then we say that $\mathcal{I}$ is a *model* of $\mathcal{T}$.

An f$_{KD}$-$\mathcal{SHIN}$ *RBox* $\mathcal{R}$ is a finite set of fuzzy role axioms. Axioms of the form $\mathsf{Trans}(R)$ are called *fuzzy transitive role axioms*, while axioms of the form $R \sqsubseteq S$ are called *fuzzy role inclusion axioms*. A fuzzy interpretation $\mathcal{I}$ satisfies an axiom $\mathsf{Trans}(R)$ if $\forall a, c \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a,c) \ge \sup_{b \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(a,b), R^{\mathcal{I}}(b,c))\}$ while it satisfies $R \sqsubseteq S$ if $\forall \langle a,b \rangle \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a,b) \le S^{\mathcal{I}}(a,b)$. Finally, $\mathcal{I}$ satisfies an f$_{KD}$-$\mathcal{SHIN}$ RBox if it satisfies every axiom in $\mathcal{R}$. In that case we say that $\mathcal{I}$ is a model of $\mathcal{R}$. A set of fuzzy role inclusion axioms defines a *role hierarchy* $\mathcal{R}_h$. Additionally, we note that the semantics of role inclusion axioms imply that if $R \sqsubseteq S$, then also $\mathsf{Inv}(R) \sqsubseteq \mathsf{Inv}(S)$, like in the classical case.

An f$_{KD}$-$\mathcal{SHIN}$ *ABox* $\mathcal{A}$ is a finite set of *fuzzy assertions* [5] of the form $(a : C) \bowtie n$ or $((a,b) : R) \bowtie n$, where $\bowtie$ stands for $\ge, >, \le$ and $<$, and $n \in [0,1]$ or of the form $a \not\doteq b$. Intuitively, a fuzzy assertion of the form $(a : C) \ge n$ means that the membership degree of the individual a to the concept C is at least equal to n. We call assertions defined using inequalities $\ge, >$ *positive*, while those using $\le, <$ *negative*. Formally, given a fuzzy interpretation $\mathcal{I}$,

$$\begin{aligned}
\mathcal{I} \text{ satisfies } (a : C) \ge n \quad &\text{if} \quad C^{\mathcal{I}}(a^{\mathcal{I}}) \ge n,\\
\mathcal{I} \text{ satisfies } (a : C) \le n \quad &\text{if} \quad C^{\mathcal{I}}(a^{\mathcal{I}}) \le n,\\
\mathcal{I} \text{ satisfies } ((a,b) : R) \ge n \quad &\text{if} \quad R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \ge n,\\
\mathcal{I} \text{ satisfies } ((a,b) : R) \le n \quad &\text{if} \quad R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \le n,\\
\mathcal{I} \text{ satisfies } a \not\doteq b \quad &\text{if} \quad a^{\mathcal{I}} \ne b^{\mathcal{I}}.
\end{aligned}$$

The satisfiability of fuzzy assertions with $>, <$ is defined analogously. Observe that, we can also simulate assertions of the form $(a : C) = n$ by considering two assertions of the form $(a : C) \ge n$ and $(a : C) \le n$. A fuzzy interpretation $\mathcal{I}$ satisfies an f$_{KD}$-$\mathcal{SHIN}$ ABox $\mathcal{A}$ iff it satisfies all fuzzy assertions in $\mathcal{A}$; in this case, we say that $\mathcal{I}$ is a model of $\mathcal{A}$.

Without loss of generality, we assume that no negative assertions exist. Negative assertions of the form $(a : C) \leq n$ and $(a : C) < n$ can be transformed into their *positive inequality normal form* (PINF), by applying a fuzzy complement in both sides getting, $(a : \neg C) \geq 1 - n$ and $(a : \neg C) > 1 - n$ (similarly with role assertions), respectively. Furthermore, we assume that a fuzzy ABox has been *normalized* [12], i.e. fuzzy assertions of the form $(a : C) > n$ are replaced by assertions of the form $(a : C) \geq n + \epsilon$, where ϵ is a small number converging to 0. Please note that in a normalized fuzzy KB with only positive inequalities degrees range over $[+\epsilon, 1 + \epsilon]$. Also note that a fuzzy ABox is consistent iff the normalized one is [13]. For a fuzzy ABox we define the set of *relative degrees* as

$$N^{\mathcal{A}} = \{0, 0.5, 1\} \cup \{1 - n, n \mid (a : C \geq n) \in \mathcal{A} \text{ or } ((a, b) : R) \geq n \in \mathcal{A}\}$$

An f$_{KD}$-$\mathcal{SHIN}$ *knowledge base* (KB) is defined as $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$. An interpretation $\mathcal{I}$ satisfies an f$_{KD}$-$\mathcal{SHIN}$ knowledge base Σ if it satisfies every axiom in $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$. In that case $\mathcal{I}$ is called a model of Σ.

Now we define the inference services of f$_{KD}$-$\mathcal{SHIN}$.

- **KB Satisfiability:** An f$_{KD}$-$\mathcal{SHIN}$ knowledge base $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$ is *satisfiable* (unsatisfiable) iff there exists (does not exist) a fuzzy interpretation $\mathcal{I}$ which satisfies all axioms in Σ.
- **Concepts n-satisfiabilty:** An f$_{KD}$-$\mathcal{SHIN}$-concept C is *n-satisfiable* w.r.t. Σ iff there exists a model $\mathcal{I}$ of Σ in which there exists some $a \in \Delta^{\mathcal{I}}$ such that $C^{\mathcal{I}}(a) = n$, and $n \in (0, 1]$.
- **Concept Subsumption:** A fuzzy concept C is subsumed by D w.r.t. Σ iff in every model $\mathcal{I}$ of Σ we have that $\forall d \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(d) \leq D^{\mathcal{I}}(d)$.
- **ABox Consistency:** An f$_{KD}$-$\mathcal{SHIN}$ $\mathcal{A}$ is *consistent* (*inconsistent*) w.r.t. a TBox $\mathcal{T}$ and an RBox $\mathcal{R}$ if there exists (does not exist) a model $\mathcal{I}$ of $\mathcal{T}$ and $\mathcal{R}$ which satisfies every assertion in $\mathcal{A}$.
- **Entailment:** Given a concept or role axiom or a fuzzy assertion, Ψ, we say that Σ *entails* Ψ, writing $\Sigma \models \Psi$ iff every model $\mathcal{I}$ of Σ satisfies Ψ.
- **Greater Lower Bound (glb):** The *greatest lower bound* of an assertion Φ w.r.t. Σ is defined as,

$$glb(\Sigma, \Phi) = \sup\{n \mid \Sigma \models \Phi \geq n\}, \text{ where } \sup \emptyset = 0.$$

As we note glb, actually consists of a set of entailment tests.

The problems of concept n-satisfiability, subsumption and entailment w.r.t. a knowledge base Σ can be reduced to the problem of knowledge base satisfiability Σ [5, 6]. Here, the reductions are slightly modified due to PINF and normalization. More precisely, a concept C is n-satisfiable w.r.t. $\mathcal{T}$ and $\mathcal{R}$ iff $\{(a : C) \geq n\}$ is consistent w.r.t. $\mathcal{T}$ and $\mathcal{R}$. Moreover, for $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$, and a PINF assertion $\phi \geq n$, where ϕ is a classical $\mathcal{SHIN}$ assertion, $\Sigma \models \phi \geq n$ iff $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \cup \{\neg\phi \geq 1 - n + \epsilon\} \rangle$ is unsatisfiable. Furthermore, $\Sigma \models C \sqsubseteq D$ iff $\langle \mathcal{T}, \mathcal{R}, \mathcal{A} \cup \{(a : C) \geq n, (a : \neg D) \geq 1 - n + \epsilon\} \rangle$ is unsatisfiable, for both $n \in \{n_1, n_2\}$, $n_1 \in (0, 0.5]$ and $n_2 \in (0.5, 1]$.

3 Reasoning with f$_{KD}$-$\mathcal{SHIN}$

In the previous section we show that all inference problems of fuzzy DLs, can be reduced to the problem of knowledge base satisfiability. Consequently, we have to construct an algorithm that decides such a reasoning problem. Our method will be based on tableaux algorithms.

Without loss of generality, we assume all concepts C occurring in assertions to be in their *negation normal form* (NNF) [3], denoted by $\sim C$; i.e., negations occur in front of concept names only. An f$_{KD}$-$\mathcal{SHIN}$-concept can be transformed into an equivalent one in NNF by pushing negations inwards making use of the De Morgan laws and the dualities between $\exists$ and $\forall$, and between concepts $\geq$ and $\leq$.

Definition 2. *For every concept D we inductively define the set of* sub-concepts *of $(sub(D))$ as,*

$$\begin{aligned}
sub(A) &= \{A\} \textit{ for every atomic concept } A \in \mathbf{C},\\
sub(C \sqcap D) &= \{C \sqcap D\} \cup \{sub(C)\} \cup \{sub(D)\},\\
sub(C \sqcup D) &= \{C \sqcup D\} \cup \{sub(C)\} \cup \{sub(D)\},\\
sub(\exists R.C) &= \{\exists R.C\} \cup \{sub(C)\},\\
sub(\forall R.C) &= \{\forall R.C\} \cup \{sub(C)\},\\
sub(\geq nR) &= \{\geq nR\}\\
sub(\leq nR) &= \{\leq nR\}
\end{aligned}$$

Definition 3. *For a fuzzy concept D and an RBox $\mathcal{R}$ we define $cl(D, \mathcal{R})$ as the smallest set of f$_{KD}$-$\mathcal{SI}$-concept which satisfies the following:*

- *$D \in cl(D, \mathcal{R})$,*
- *$cl(D, \mathcal{R})$ is closed under sub-concepts of D and $\sim D$, and*
- *if $\forall R.C \in cl(D, \mathcal{R})$ and $\mathsf{Trans}(P)$ with $P \sqsubseteq^* R$, then $\forall P.C \in cl(D, \mathcal{R})$*

Finally we define $cl(\mathcal{A}, \mathcal{R}) = \bigcup_{(a:D) \geq n \in \mathcal{A}} cl(D, \mathcal{R})$.

When $\mathcal{R}$ is clear from the context we simply write $cl(\mathcal{A})$.

Definition 4. *If $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$ is an f$_{KD}$-$\mathcal{SHIN}$ knowledge base, $\boldsymbol{R}_{\mathcal{A}}$ is the set of roles occurring in Σ together with their inverses, $\mathbf{I}_{\mathcal{A}}$ is the set of individuals in $\mathcal{A}$, a fuzzy tableau T for Σ is defined to be a quadruple ($\boldsymbol{S}$, $\mathcal{L}$, $\mathcal{E}$, $\mathcal{V}$) such that: $\boldsymbol{S}$ is a set of elements, $\mathcal{L} : \boldsymbol{S} \times cl(\mathcal{A}) \to [0,1]$ maps each element and concept to the membership degree of that element to the concept, $\mathcal{E} : \boldsymbol{R}_{\mathcal{A}} \times \boldsymbol{S} \times \boldsymbol{S} \to [0,1]$ maps each role of $\boldsymbol{R}_{\mathcal{A}}$ and pair of elements to the membership degree of the pair to the role, and $\mathcal{V} : \mathbf{I}_{\mathcal{A}} \to \boldsymbol{S}$ maps individuals occurring in $\mathcal{A}$ to elements of $\boldsymbol{S}$. For all $s, t \in \boldsymbol{S}$, $C, E \in cl(\mathcal{A})$, $n \in [0,1]$ and $R \in \boldsymbol{R}_{\mathcal{A}}$, T satisfies:*

1. *$\mathcal{L}(s, \bot) = 0$ and $\mathcal{L}(s, \top) = 1$ for all $s \in \boldsymbol{S}$,*

2. *If* $\mathcal{L}(s, \neg A) = n$, *then* $\mathcal{L}(s, A) = 1 - n$,
3. *If* $\mathcal{E}(\neg R, \langle s, t\rangle) = n$, *then* $\mathcal{E}(R, \langle s, t\rangle) = 1 - n$,
4. *If* $\mathcal{L}(s, C \sqcap E) \geq n$, *then* $\mathcal{L}(s, C) \geq n$ *and* $\mathcal{L}(s, E) \geq n$,
5. *If* $\mathcal{L}(s, C \sqcup E) \geq n$, *then* $\mathcal{L}(s, C) \geq n$ *or* $\mathcal{L}(s, E) \geq n$,
6. *If* $\mathcal{L}(s, \forall R.C) \geq n$, *then either* $\mathcal{E}(\neg R, \langle s, t\rangle) \geq n$ *or* $\mathcal{L}(t, C) \geq n$,
7. *If* $\mathcal{L}(s, \exists R.C) \geq n$, *then there exists* $t \in \boldsymbol{S}$ *such that* $\mathcal{E}(R, \langle s, t\rangle) \geq n$ *and* $\mathcal{L}(t, C) \geq n$,
8. *If* $\mathcal{L}(s, \forall R.C) \geq n$, *then either* $\mathcal{E}(\neg P, \langle s, t\rangle) \geq n$, *for* $P \mathrel{\underline{\sqsubseteq}^*} R$ *with* $\mathsf{Trans}(P)$ *or* $\mathcal{L}(t, \forall P.C) \geq n$,
9. $\mathcal{E}(R, \langle s, t\rangle) \geq n$ *iff* $\mathcal{E}(\mathsf{Inv}(R), \langle t, s\rangle) \geq n$,
10. *If* $\mathcal{E}(R, \langle s, t\rangle) \geq n$ *and* $R \mathrel{\underline{\sqsubseteq}^*} S$, *then* $\mathcal{E}(S, \langle s, t\rangle) \geq n$,
11. *If* $\mathcal{L}(s, \geq pR) \geq n$, *then* $\sharp R^T(s, \geq, n) \geq p$,
12. *If* $\mathcal{L}(s, \leq pR) \geq n$, *then* $\sharp R^T(s, \geq, 1 - n + \epsilon) \leq p$,
13. *If* $C \sqsubseteq D \in \mathcal{T}$, *then either* $\mathcal{L}(s, C) \geq 1 - n + \epsilon$ *or* $\mathcal{L}(s, D) \geq n$, *for all* $s \in \boldsymbol{S}$ *and* $n \in N^{\mathcal{A}}$,
14. *If* $(a : C) \geq n \in \mathcal{A}$, *then* $\mathcal{L}(\mathcal{V}(a), C) \geq n$,
15. *If* $((a, b) : R) \geq n \in \mathcal{A}$, *then* $\mathcal{E}(R, \langle \mathcal{V}(a), \mathcal{V}(b)\rangle) \geq n$,
16. *If* $a \neq b \in \mathcal{A}$, *then* $\mathcal{V}(a) \neq \mathcal{V}(b)$.

where $\sharp$ *denotes the cardinality of a set,* $R^T(s, \geq, n) = \{t \in \boldsymbol{S} \mid \mathcal{E}(R, \langle s, t\rangle) \geq n\}$ *returns the set of elements* $t \in \boldsymbol{S}$ *that participate in* R *with some element* s *with a degree, greater or equal or greater than a given degree* n.

Lemma 1. *An* f_{KD}-$\mathcal{SHIN}$ *knowledge base* Σ *is satisfiable iff there exists a fuzzy tableau for* Σ.

For a detailed proof of the above lemma as well as the intuition behind the properties of Definition 3 the reader is referred to [6] and [12].

The above lemma establishes a connection between the satisfiability of a knowledge base (existence of a model) and the existence of a fuzzy tableaux for Σ. Thus, it suggests that in order to decide the key inference problems of f_{KD}-$\mathcal{SHIN}$ we have to develop an algorithm that given an f_{KD}-$\mathcal{SHIN}$ KB Σ it constructs a fuzzy tableau for Σ.

3.1 The Tableaux Algorithm

In order to decide knowledge base satisfiability a procedure that constructs a fuzzy tableau for an f_{KD}-$\mathcal{SHIN}$ knowledge base has to be determined. In the current section we will provide the technical details for such an algorithm.

Definition 5. *A completion-forest* $\mathcal{F}$ *for an* f_{KD}-$\mathcal{SHIN}$ *knowledge base is a collection of trees whose distinguished roots are arbitrarily connected by edges. Each node* x *is labelled with a set* $\mathcal{L}(x) = \{\langle C, \geq, n\rangle\}$, *where* $C \in cl(\mathcal{A})$ *and*

$n \in [+\epsilon, 1+\epsilon]$. *Each edge* $\langle x, y\rangle$ *is labelled with a set* $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \geq, n\rangle\}$, *where* $S := R \mid \neg R$, *and* $R \in \boldsymbol{R}_{\mathcal{A}}$ *is a (possibly inverse) role occurring in* $\mathcal{A}$.

If nodes x *and* y *are connected by an edge* $\langle x, y\rangle$ *with* $\langle P, \geq, n\rangle \in \mathcal{L}(\langle x, y\rangle)$, *and* $P \sqsubseteq^* R$, *then* y *is called an* $R_{\geq n}$-successor *of* x *and* x *is called an* $R_{\geq n}$-predecessor *of* y. *If* y *is an* $R_{\geq n}$*-successor or an* $\mathsf{Inv}(R)_{\geq n}$-predecessor *of* x, *then* y *is called an* $R_{\geq n}$*-neighbour of* x. *Let* y *be an* $R_{>n}$*-neighbour of* x. *Then, the edge* $\langle x, y\rangle$ *is* conjugated *with triples* $\langle \neg R, \geq, m\rangle$ *if* $n + m \geq 1$. *Similarly, we can extend it to the case of* $R_{\geq n}$*-neighbours. As usual,* ancestor *is the transitive closure of* predecessor.

For two roles P, R, *a node* x *in* $\mathcal{F}$, *an inequality* $\geq$ *and a membership degree* $n \in [0, 1]$ *we define:* $R_C^{\mathcal{F}}(x, \geq, n) = \{y \mid y$ *is an* $R_{\geq n'}$*-neighbour of* x, *and* $\langle x, y\rangle$ *is conjugated with* $\langle \neg R, \geq, n\rangle\}$.

A node x *is* blocked *iff it is not a root node and it is either directly or indirectly blocked. A node* x *is* directly blocked *iff none of its ancestors is blocked, and it has ancestors* x', y *and* y' *such that:*

1. y *is not a root node,*
2. x *is a successor of* x' *and* y *a successor of* y',
3. $\mathcal{L}(x) = \mathcal{L}(y)$ *and* $\mathcal{L}(x') = \mathcal{L}(y')$ *and,*
4. $\mathcal{L}(\langle x', x\rangle) = \mathcal{L}(\langle y', y\rangle)$.

In this case we say that y *blocks* x. *A node* y *is indirectly blocked iff one of its ancestors is blocked, or it is a successor of a node* x *and* $\mathcal{L}(\langle x, y\rangle) = \emptyset$.

For a node x, $\mathcal{L}(x)$ *is said to contain a clash if it contains one of the following:*

- *two conjugated pairs of triples,*
- *one of* $\langle \bot, \geq, n\rangle$, *with* $n > 0$ *or* $\langle C, \geq, 1+\epsilon\rangle$, *or*
- *some triple* $\langle \leq pR, \geq, n\rangle$ *and* x *has* $p+1$ $R_{\geq n_i}$*-neighbours* $y_0, \ldots, y_p$, $\langle x, y_i\rangle$ *is conjugated with* $\langle \neg R, \geq, n\rangle$ *and* $y_i \neq y_j$, $n_i, n \in [0, 1]$, *for all* $0 \leq i < j \leq p$

Moreover, for an edge $\langle x, y\rangle$, $\mathcal{L}(\langle x, y\rangle)$ *is said to contain a clash if (i) it contains two conjugated triples, or (ii) it contains the triple* $\langle R, \geq, 1+\epsilon\rangle$, *or (iii)* $\mathcal{L}(\langle x, y\rangle) \cup \{\langle \mathsf{Inv}(R), \geq, n\rangle \mid \langle R, \geq, n\rangle \in \mathcal{L}(\langle y, x\rangle)\}$, *where* x, y *are root nodes, contains two conjugated triples.*

For an f_{KD}-$\mathcal{SHIN}$ *knowledge base, the algorithm initialises a forest* $\mathcal{F}$ *to contain*

i. *a root node* x_{a_i}, *for each individual* $a_i \in \mathbf{I}_{\mathcal{A}}$ *occurring in the ABox* $\mathcal{A}$, *labelled with* $\mathcal{L}(x_{a_i})$ *such that:* $\mathcal{L}(x_{a_i}) = \{\langle C, \geq, n\rangle \mid (a_i : C) \geq n \in \mathcal{A}\}$,

ii. *an edge* $\langle x_{a_i}, x_{a_j}\rangle$, *for each assertion* $((a_i, a_j) : R) \geq n \in \mathcal{A}$, *labelled with* $\mathcal{L}(\langle x_{a_i}, x_{a_j}\rangle)$ *such that:* $\mathcal{L}(\langle x_{a_i}, x_{a_j}\rangle) = \{\langle R, \geq, n\rangle \mid \langle R, \geq, n\rangle \in \mathcal{A}\}$,

iii. *the relation* $\not\doteq$ *as* $x_{a_i} \not\doteq x_{a_j}$ *if* $a_i \not\doteq a_j \in \mathcal{A}$ *and the relation* $\doteq$ *to be empty.*

Finally, the algorithm expands $\mathcal{R}$ *by adding role inclusion axioms* $\mathsf{Inv}(P) \sqsubseteq \mathsf{Inv}(R)$, *for all* $P \sqsubseteq R \in \mathcal{R}$ *and by adding* $\mathsf{Trans}(\mathsf{Inv}(R))$ *for all* $\mathsf{Trans}(R) \in \mathcal{R}$.

Table 2. Expansion rules for f$_{KD}$-$\mathcal{SHIN}$

Rule	Description
$\sqcap$	if 1. $\langle C_1 \sqcap C_2, \geq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and 2. $\{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\} \not\subseteq \mathcal{L}(x)$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\}$
$\sqcup$	if 1. $\langle C_1 \sqcup C_2, \geq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and 2. $\{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ for some $C \in \{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\}$
$\exists$	if 1. $\langle \exists R.C, \geq, n\rangle \in \mathcal{L}(x)$, x is not blocked, and 2. x has some $R_{\geq n}$-neighbour y with $\langle C, \geq, n\rangle \in \mathcal{L}(y)$ then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \geq, n\rangle\}$, $\mathcal{L}(y) = \{\langle C, \geq, n\rangle\}$
$\forall$	if 1. $\langle \forall R.C, \geq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, 2. x has an $R_{\geq' n'}$-neighbour y with $\langle C, \geq, n\rangle \notin \mathcal{L}(y)$ and 3. $\langle x, y\rangle$ conjugates with $\langle \neg R, \geq, n\rangle$ then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle C, \geq, n\rangle\}$
$\forall_+$	if 1. $\langle \forall S.C, \geq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, 2. there exists some role R, with $\mathsf{Trans}(R)$ and $R \sqsubseteq^{*} S$, 3. x has an $R_{\geq' n'}$-neighbour y with $\langle \forall R.C, \geq, n\rangle \notin \mathcal{L}(y)$, and 4. $\langle x, y\rangle$ conjugates with $\langle \neg R, \geq, n\rangle$ then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle \forall R.C, \geq, n\rangle\}$
$\geq$	if 1. $\langle \geq pR, \geq, n\rangle \in \mathcal{L}(x)$, x is not blocked, 2. there are no p $R_{\geq, n}$-neighbours $y_1, \ldots, y_p$ of x with $y_i \neq y_j$ for $1 \leq i < j \leq p$ then create p new nodes $y_1, \ldots, y_p$, with $\mathcal{L}(\langle x, y_i\rangle) = \{\langle R, \geq, n\rangle\}$ and $y_i \neq y_j$ for $1 \leq i < j \leq p$
$\leq$	if 1. $\langle \leq pR, \geq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, 2. $\sharp R_C^{\mathcal{F}}(x, \geq, n) > p$, there are two of them y, z, with no $y \neq z$ and 3. y is neither a root node nor an ancestor of z then 1. $\mathcal{L}(z) \rightarrow \mathcal{L}(z) \cup \mathcal{L}(y)$ and 2. if z is an ancestor of x then $\mathcal{L}(\langle z, x\rangle) \longrightarrow \mathcal{L}(\langle z, x\rangle) \cup \mathsf{Inv}(\mathcal{L}(\langle x, y\rangle))$ else $\mathcal{L}(\langle x, z\rangle) \longrightarrow \mathcal{L}(\langle x, z\rangle) \cup \mathcal{L}(\langle x, y\rangle)$ 3. $\mathcal{L}(\langle x, y\rangle) \longrightarrow \emptyset$ and set $u \neq z$ for all u with $u \neq y$
$\leq_r$	if 1. $\langle \leq pR, \geq, n\rangle \in \mathcal{L}(x)$, 2. $\sharp R_C^{\mathcal{F}}(x, \geq, n) > p$, there are two of them y, z, both root nodes, with no $y \neq z$ and then 1. $\mathcal{L}(z) \rightarrow \mathcal{L}(z) \cup \mathcal{L}(y)$ and 2. For all edges $\langle y, w\rangle$: i. if the edge $\langle z, w\rangle$ does not exist, create it with $\mathcal{L}(\langle z, w\rangle) = \emptyset$ ii. $\mathcal{L}(\langle z, w\rangle) \longrightarrow \mathcal{L}(\langle z, w\rangle) \cup \mathcal{L}(\langle y, w\rangle)$ 3. For all edges $\langle w, y\rangle$: i. if the edge $\langle w, z\rangle$ does not exist, create it with $\mathcal{L}(\langle w, z\rangle) = \emptyset$ ii. $\mathcal{L}(\langle w, z\rangle) \longrightarrow \mathcal{L}(\langle w, z\rangle) \cup \mathcal{L}(\langle w, y\rangle)$ 4. Set $\mathcal{L}(y) = \emptyset$ and remove all edges to/from y 5. Set $u \neq z$ for all u with $u \neq y$ and set $y \doteq z$
$\sqsubseteq$	if 1. $C \sqsubseteq D \in \mathcal{T}$, x is not indirectly blocked, and 2. $\{\langle \neg C, \geq, 1 - n + \epsilon\rangle, \langle D, \geq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$ for $n \in N^{\mathcal{A}}$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{E\}$ for some $E \in \{\langle \neg C, \leq, 1 - n + \epsilon\rangle, \langle D, \geq, n\rangle\}$

$\mathcal{F}$ is then expanded by repeatedly applying the completion rules from Table 2. The completion-forest is complete when, for some node x, $\mathcal{L}(x)$ contains a clash, or none of the completion rules is applicable. The algorithm stops when a clash occurs; it answers 'Σ is satisfiable' iff the completion rules can be applied in such a way that they yield a complete and clash-free completion-forest, and 'Σ is unsatisfiable' otherwise.

Lemma 2. *Let Σ be an f_{KD}-$\mathcal{SHIN}$ knowledge base. Then*

1. *when started for Σ the tableaux algorithm terminates*
2. *Σ has a fuzzy tableau if and only if the expansion rules can be applied to Σ such that they yield a complete and clash-free completion forest.*

Finally, we conclude this section with an illustrative example that shows how the tableaux algorithm works.

Example 1. Consider the knowledge base $\Sigma = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$ where:

$$\begin{aligned}&\mathcal{T} = \{\mathsf{Arm} \sqsubseteq \exists \mathsf{isPartOf.Body},\ \mathsf{Body} \sqsubseteq \exists \mathsf{isPartOf.Human}\}\\ &\mathcal{R} = \{\mathsf{Trans(isPartOf)}\}\\ &\mathcal{A} = \{((\mathsf{o}_1, \mathsf{o}_2) : \mathsf{isPartOf}) \geq 0.8,\ ((\mathsf{o}_2, \mathsf{o}_3) : \mathsf{isPartOf}) \geq 0.9,\\ &\qquad (\mathsf{o}_2 : \mathsf{Body}) \geq 0.85,\ (\mathsf{o}_1 : \mathsf{Arm}) \geq 0.75\}\end{aligned}$$

Now we want to use our reasoning algorithm to see if

$$\Sigma \models (\mathsf{o}_3 : \exists \mathsf{Inv(isPartOf).Body} \sqcap \exists \mathsf{Inv(isPartOf).Arm}) < 0.75.$$

First we transform this negative assertion into its equivalent PINF form and then into its NNF form having finally the assertion $(\mathsf{o}_3 : \forall \mathsf{Inv(isPartOf)}.\neg\mathsf{Body} \sqcup \forall \mathsf{Inv(isPartOf)}.\neg\mathsf{Arm}) > 0.25$. Subsequently, entailment checking is reduced to consistency of $\mathcal{A}' = \mathcal{A} \cup \{(\mathsf{o}_3 : \forall \mathsf{Inv(isPartOf)}.\neg\mathsf{Body} \sqcup \forall \mathsf{Inv(isPartOf)}.\neg\mathsf{Arm}) > 0.25\}$, w.r.t. $\mathcal{R}$ and $\mathcal{T}$. According to Definition 5 the algorithm initializes a completion-forest to contain the following triples:

$$\begin{aligned}&(1)\ \langle \mathsf{isPartOf}, \geq, 0.8\rangle \in \mathcal{L}(\langle x_{\mathsf{o}_1}, x_{\mathsf{o}_2}\rangle)\\ &(2)\ \langle \mathsf{isPartOf}, \geq, 0.9\rangle \in \mathcal{L}(\langle x_{\mathsf{o}_2}, x_{\mathsf{o}_3}\rangle)\\ &(3)\ \langle \mathsf{Body}, \geq, 0.85\rangle \in \mathcal{L}(x_{\mathsf{o}_2})\\ &(4)\ \langle \mathsf{Arm}, \geq, 0.75\rangle \in \mathcal{L}(x_{\mathsf{o}_1})\\ &(5)\ \langle \forall \mathsf{isPartOf}^-.\neg\mathsf{Body} \sqcup \forall \mathsf{isPartOf}^-.\neg\mathsf{Arm}, >, 0.25\rangle \in \mathcal{L}(x_{\mathsf{o}_3})\end{aligned}$$

Furthermore, the algorithm expands $\mathcal{R}$ by adding the axiom $\mathsf{Trans(isPartOf^-)}$. Subsequently, by applying expansion rules from Table 2 we have the following steps:

$$(6)\ \langle \forall \mathsf{isPartOf}^-.\neg\mathsf{Body}, >, 0.25\rangle \in \mathcal{L}(x_{\mathsf{o}_3}) | \langle \forall \mathsf{isPartOf}^-.\neg\mathsf{Arm}, >, 0.25\rangle \in \mathcal{L}(x_{\mathsf{o}_3})\ \sqcup$$

Hence at this point we have two possible completion forests. For the first one we have:

(6_1) $\langle \forall \mathsf{isPartOf}^-.\neg \mathsf{Body}, >, 0.25 \rangle \in \mathcal{L}(x_{\mathsf{o}_3})$
(7_1) $\langle \neg \mathsf{Body}, >, 0.25 \rangle \in \mathcal{L}(x_{\mathsf{o}_2})$ $\quad \forall : (6_1), (2)$
(8_1) clash (7_1) and (3)

while for the second possible completion-forest we have:

(6_2) $\langle \forall \mathsf{isPartOf}^-.\neg \mathsf{Arm}, >, 0.25 \rangle \in \mathcal{L}(x_{\mathsf{o}_3})$
(7_2) $\langle \neg \mathsf{Arm}, >, 0.25 \rangle \in \mathcal{L}(x_{\mathsf{o}_2})$ $\quad \forall : (6_2), (2)$
(8_2) $\langle \forall \mathsf{isPartOf}^-.\mathsf{Arm}, <, 0.75 \rangle \in \mathcal{L}(x_{\mathsf{o}_2})$ $\quad \forall_+ : (6_2), (2)$
(9_2) $\langle \neg \mathsf{Arm}, >, 0.25 \rangle \in \mathcal{L}(x_{\mathsf{o}_1})$ $\quad \forall : (8_2), (1)$
(10_2) clash (9_2) and (4)

Thus, since all possible expansions result to a clash, $\mathcal{A}'$ is inconsistent and the knowledge base entails the fuzzy assertion.

4 FiRE: A Prototype $\mathbf{f}_{KD}$-$\mathcal{SHIN}$ Reasoning System

FiRE is a JAVA implementation of a fuzzy DL reasoning engine for vague knowledge. Currently it implements the tableaux reasoning algorithm for f_{KD}-$\mathcal{SHIN}$ we presented in the previous section. Apart from the f_{KD}-$\mathcal{SHIN}$ reasoner, FiRE is also able to serialize a fuzzy KB into RDF triples and store it in the Sesame RDF triple store [14]. Then it is able to query Sesame using very expressive fuzzy conjunctive query languages [11]. In this section the graphical user interface, the syntax and the inference services of FiRE are briefly introduced.

4.1 FiRE Interface

FiRE can be found at `http://www.image.ece.ntua.gr/~nsimou/FiRE` together with installation instructions and examples. Figure 1 depicts the main GUI of FiRE. Its user interface consists of the *editor panel*, the *inference services panel* and the *output panel*. The user can create or edit an existing fuzzy knowledge base using the editor panel. The inference services panel allows the user to make different kinds of queries to the knowledge base (entailment, subsumption and glb) and also to query a Sesame repository using fuzzy conjunctive queries [11]. Finally, the output panel consists of four different tabs, each one displaying information depending on the user operation, like a trace of the tableaux expansion, possible syntax errors of the KB, classification of the KB (computing the subsumption hierarchy), and more.

4.2 FiRE Syntax

The current version of FiRE is using the Knowledge Representation System Specification (KRSS) proposal[3]. Since as we show in the previous sections we

[3] `http://dl.kr.org/krss-spec.ps`

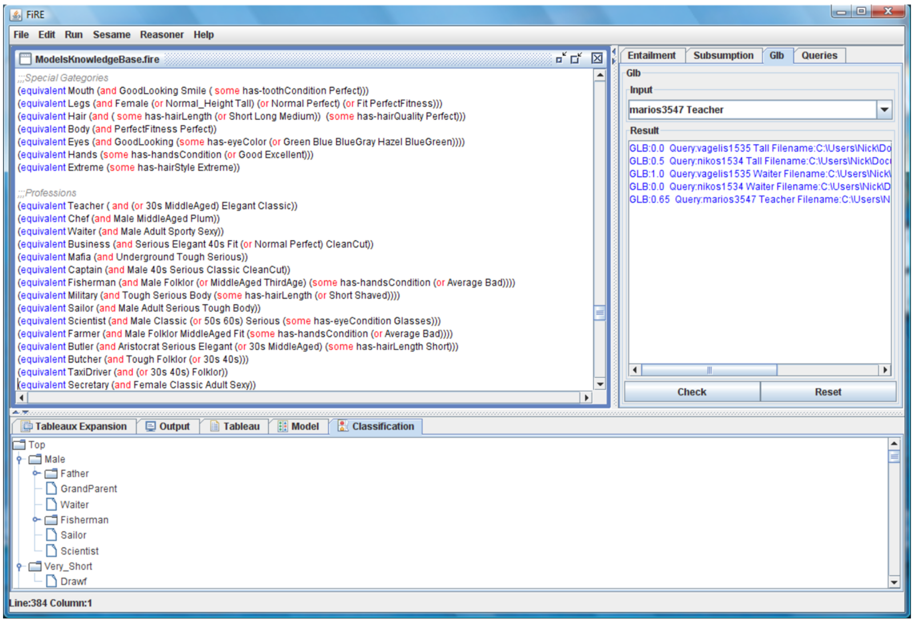

Fig. 1. The FiRE user interface: the editor panel (upper left), the inference services panel (upper right) and the output panel (bottom)

impose no syntactic changes to concept and role axioms, a user is capable of specifying concept and roles axioms using the standard KRSS syntax. So for example, one can define the concept MiddleAged with the following axiom:

```
(complete MiddleAged (or Forties Fifties))
```

using the keywords **complete** for specifying equivalence ($\equiv$) and **or** for specifying disjunction ($\sqcup$). Similarly we can specify subsumption axioms using the keyword **implies** or role axioms using the keywords **transitive**, **parent** and **inverse** for transitive role axioms, role inclusion axioms or specifying the inverse of a role, respectively.

On the other hand individual axioms (assertion) of KRSS need to be extended in order to capture confidence degrees. More precisely, fuzzy concept and role assertions are specified by using the following patterns:

```
(instance ind Concept ineqType n)
(related ind1 ind2 Role ineqType n)
```

where ineqType is one of ">=", ">", "<=", "<", "=", and $n \in (0, 1]$ is a degree. Thus, in the first syntax we use the keyword **instance** to declare a fuzzy assertion between an individual and a concept with some inequality type and degree n; similarly with role assertions and keyword **related**.

Example 2. The syntax of the assertions $alice : \mathsf{Female}$, $(paul : (\mathsf{Tall} \sqcap \mathsf{Thin}) \geq 0.8)$ and $((frank, paul) : \mathsf{has-friend}) \geq 0.7$ are shown below in FiRE syntax.

```
(instance alice Female)
(instance paul (and Tall Thin) > 0.8)
(related frank paul has-friend >= 0.7)
```

4.3 Inference Services

FiRE offers all the fuzzy DL inference services we introduced in Section 2 plus a *global glb* service and answering *conjunctive queries* over RDF repositories, described below. More precisely, it allows to check ABox consistency. If the ABox is consistent w.r.t. to a TBox and an RBox, FiRE provides the user with a sample model of the knowledge base in the Model tab of the output panel. If the ABox is not consistent then a "`not satisfiable`" message is reported in the tableaux tab.

Then, FiRE offers a number of specialized tabs in the inference services panel that implement many services. More precisely, it offers an *Entailment* inference tab that allows users to ask for the entailment of fuzzy assertions. The syntax for such queries is the same as the syntax of specifying concept assertions. For example, in order to check whether $\Sigma \models (a : C) \geq n$ the user should enter the statement `instance a C >= n` in the entailment tab. On the other hand subsumption queries are specified in the *Subsumption* inference tab. Their syntax is of the following form `(concept1) (concept2)` where concept1 and concept2 are f_{KD}-$\mathcal{SHIN}$-concepts.

Subsequently, FiRE offers for computing the glb of an individual to a concept w.r.t. a knowledge base Σ. Glb queries are evaluated by FiRE performing entailment queries for all the degrees contained in the ABox, using the binary search algorithm in order to reduce the entailment tests. The syntax of glb queries is of the form `individual (concept)` where concept can be an f_{KD}-$\mathcal{SHIN}$-concept. Besides glb queries, FiRE offers for computing the *global glb* of a knowledge base. More precisely, it computes the glb of all the individuals in the ABox with all the defined concepts of the TBox. Roughly speaking, this process materializes (almost) all the relevant implied knowledge that is entailed by the knowledge base, i.e. the one that involves the defined concepts.

Finally, besides the standard inference services of fuzzy DLs, FiRE also offers the *Queries* inference tab, which can be used in order to issue expressive fuzzy conjunctive queries over a Sesame repository. More precisely, the user can issue *conjunctive threshold queries* (CTQs) or *generalized fuzzy conjunctive queries* (GFCQs), like *fuzzy threshold queries*, *fuzzy aggregation queries* and *fuzzy weighted t-norm queries*, as these have been defined and implemented for fuzzy-DL-Lite in [11]. An example GFCQ is the following:

```
x <- Goodlooking(x):0.6 ^ has-hairLength(x,y):1 ^ Long(y):0.8
```

asking for all x that are good looking and have long hair. We see that in such queries the user is capable of also specifying weights in query atoms.

(*a*) (*b*)

Fig. 2. Input image (left) and its segmentation (right)

5 Two Usage Scenarios

In the current Section we will present two application scenarios where we have tested FiRE and its potentials.

5.1 Multimedia Analysis and Scene Interpretation

One of the main research problems in multimedia analysis is how one could extract and represent all the underlying information and knowledge that exist within an image or a video. For example, an image could depict an event, a landscape, people, etc. that need to be represented in order for end-users to be able to query about them. Manual annotation is obviously very difficult and expensive hence (semi)automatic ways are explored. First, we apply image analysis algorithms, which are based on color, texture and shape criteria to group pixels and create segments which (possibly) depict an object. Subsequently, we apply a recognition system which ideally would be able to assign a semantic label to each region. Generally, this task is very difficult since moving from low-level features to high-level semantic descriptions, like complex objects is far from trivial. For those reasons proposals for knowledge-based multimedia analysis have been proposed [15, 16]. Using DLs one can provide definitions of high-level concepts and events that might exist in an image or video in order to assist the process of recognition. For example, we could have the following DL axioms:

$$\begin{array}{rcl}
\mathsf{Leaves} & \equiv & \mathsf{GreenColored} \\
\mathsf{Tree} & \equiv & \mathsf{BrownColored} \sqcap \exists \mathsf{isConnected.Leaves} \\
\mathsf{MuddyRoad} & \equiv & \mathsf{BrownColored} \sqcap \mathsf{CoarseTextured}
\end{array}$$

Image analysis is generally a process that involves a huge amount of uncertain and vague knowledge, hence we would prefer to use extended frameworks like fuzzy DLs as the underlying logical framework. Consider for example Figure 2(a) which shows a sample input image, while Figure 2(b) shows its segmentation. We see that the algorithm has identified several regions in the image for which we

Table 3. Semantic labelling

Region	Extracted Concept	Degree	Inferred Concept	Degree
$region_1$	GreenColored	0.80	Leaves	0.80
$region_2$	LightGreenColored	0.78	Grass	0.78
$region_3$	LightGreenColored	0.71	Grass	0.71
$region_4$	BrownColored	0.69	MuddyRoad	0.69
	CoarseTextured	0.80		
$region_5$	CoarseTextured	0.30	ClayRoad	0.30
	LightBrownColored	0.85		
$region_6$	BrownColored	0.67	MuddyRoad	0.67
	CoarseTextured	0.80		
$region_7$	LightGrayColored	0.72	TarRoad	0.70
	SmoothTextured	0.70		

can extract their MPEG-7 visual descriptors.[4] These are numerical values which provide information about the texture, shape and color of a region. Obviously, these values are very low-level and provide no semantic information. Nevertheless, one could use them in order to *move* from low-level descriptions to more high-level ones. For example, if reg_1's green component in the RGB color model was equal to 243, we can be based on a mapping (fuzzy partition) function [17] and deduce that reg_1 is GreenColored to a degree at least 0.8. On the other hand another region with a green component of 200 could be GreenColored to a degree 0.77. Similarly, we can extract additional fuzzy assertions using other MPEG-7 descriptors, like texture or shape. For example, in the leftmost part of Table 3 we see some fuzzy assertions extracted for a specific region and a concept, using MPEG-7 descriptors. Subsequently, we can use FiRE's global glb service in order to extract all the implied knowledge for the specific image [18]. The inferred assertions are depicted in the rightmost part of Table 3. We see that fuzzy DL reasoning can be used to provide more sophisticated labelling, but please note that these are still some *very* preliminary results and the current example is by no means complete.

5.2 Knowledge Based Information Retrieval and Recommendation

FiRE has been applied in an industrial strength Use Case scenario from a Greek National project. In this Use Case scenario we consider a production company, which has a knowledge base that consists of videos and images about persons (which usually are actors or models). This company wants to publish its content on the (Semantic) Web so as other advertisement or production companies can use this knowledge base to look for persons to be employed for advertisements (casting). Each entry in the knowledge base contains a photo or a video, and some specific information like body and face characteristics, age or profession-like characteristic. A user can query the knowledge base providing

[4] `http://www.chiariglione.org/mpeg/standards/mpeg-7/mpeg-7.htm`

information like the name, the height, the type of the hair, the body, age range, and more.

Usually casting people want to query such a knowledge base using some high level concepts like "Thirties", "MiddleAged", "Teen", "Kid", "Slim", "Tall", "StudentLooking", "TeacherLooking" and more, which can be used in commercials of respective context. Obviously, most of these concepts are vague (fuzzy) as for example the concepts of middle aged or tall persons cannot be precisely defined. In order to tackle the above Use Case scenario we have followed the next steps [18]:

1. *Database (DB) fuzzification:* First, we fuzzify fields of the database, in order to provide symbolic information from the existing numerical one. For example, the "age" field provides very low level information which can be used in order to define (fuzzy) concepts, like "Teen", "Twenties", "Thirties", "Old" etc. These concepts are defined as functions (fuzzy sets) that map the age value of a person a to the membership degree of a to them. Thus, we can crate fuzzy assertions. For example, the DB has that $john180$ is 34 years old, thus the function of "Thirties" tells us that $john180 : \mathsf{Thirties} \geq 0.6$.

2. *Ontology construction:* Using the above concepts, together with additional ones of our domain, we can construct an ontology for human actors (models) focusing on appearance, that is important for casting tasks. For example, we can define the concept of student looking, tall child and scientist as:

$$\begin{aligned}
\mathsf{StudentLooking} &\sqsubseteq \mathsf{Kid} \sqcup \mathsf{Teen} \\
\mathsf{TallChild} &\sqsubseteq \mathsf{Child} \sqcap (\mathsf{Short} \sqcup \mathsf{Normal_Height}) \\
\mathsf{Scientist} &\sqsubseteq \mathsf{Male} \sqcap \mathsf{Classic} \sqcap (\mathsf{50s} \sqcup \mathsf{60s}) \sqcap \\
&\qquad \mathsf{Serious} \sqcap \exists \mathsf{has\text{-}eyeCondition.Glasses}
\end{aligned}$$

   using already defined concepts. Please note that if we hadn't created the fuzzy concepts Kid, Teen, Child, Short and Normal_Height in step 1, which initially did not exist in the database, we would not be able to define the above concepts. Similarly, we can define more concepts, like GrandParent, FishermanLooking and more.

3. *Extracting implied knowledge:* The ontology together with the fuzzy assertions that are produced by step 1, due to fuzzification, as well as the crisp assertions that exist in the database (e.g. $john180$ is a Male, Latin, etc.) is loaded into FiRE. Then we compute the global glb of the knowledge base in order to extract implied knowledge. Subsequently, knowledge is serialized and stored into Sesame.

4. *Querying the KB:* Finally, end-users can issue very expressive fuzzy conjunctive queries over Sesame through the FiRE platform in order to retrieve actors. For example, for a TV commercial for hair dyes one might want to retrieve all female models, that are in their twenties, have long, good quality hair and nice eyes, or for an MP3 player commercial one might want a student looking model.

6 Discussion and Open Problems

It has been widely approved that fuzzy DLs could play an important role in the Semantic Web by serving as a mathematical framework for knowledge representation and reasoning in applications that face vague knowledge, like image analysis and understanding [19], ontology searching [11], semantic portals [20] multimedia retrieval [21] and negotiation [22]. But still the full potential of fuzzy DLs has not been exhaustively explored, since they could be used in a wealth of tasks and applications in order to enhance automation and handle degrees of confidence, membership and truth that emerge by matching, retrieval, recommendation, negotiation or recognition systems.

After the first ideas about extending classical two-valued Description Logics with fuzzy Set Theory, by Yen in [23], Tresp and Molitor [24] and Straccia [5], there has been an increasing research effort on fuzzy Description Logics. The last couple of years research is focused on providing reasoning support for very expressive fuzzy DLs, in order to support reasoning in a full fuzzy extension of the OWL web ontology language. Towards this direction, recently Stoilos et. al. [6] presented a reasoning algorithm for the fuzzy DLs f_{KD}-$\mathcal{SI}$ and f_{KD}-$\mathcal{SHIN}$, while also in another work Stoilos et. al. [12] presented an algorithm for reasoning with General Concept Inclusion axioms, which was an open problem in fuzzy DLs. Interestingly, these results gave rise to the FiRE fuzzy DL systems, presented in section 4 (also a preliminary version was reported in [8]). Furthermore, the study of reasoning algorithms for fuzzy DLs that use other norm operation is also beginning to flourish, although still most results are focused on rather basic DLs like $\mathcal{ALC}$. More precisely, Straccia [25] presented an algorithm for f_L-$\mathcal{ALC}(\mathcal{D})$, and recently Bobillo and Straccia [26] a reasoning algorithm for f_P-$\mathcal{ALC}f\,(\mathcal{D})$ ($\mathcal{ALC}$ with functional role axioms). Also these algorithms are supported by the *fuzzyDL* system [27].

On the other hand, a recent trend in DL research is mainly focused in studying efficient and scalable (tractable) Description Logics, compared to the NEXPTIME-complete OWL DL. Following this trend Straccia proposed a fuzzy extension of DL-Lite [28]. DL-Lite [29] is an interesting lightweight ontology language, since it can answer conjunctive queries in a very efficient way, by using existing database technologies. Later Pan et al. [11] proposed some very expressive extensions to the conjunctive queries of f-DL-Lite. The algorithms for these queries were implemented in the system ONTOSEARCH2[5] and evaluation showed that these can still be answered in a very efficient way. Other interesting tractable DLs are those of the $\mathcal{EL}$ family, like $\mathcal{EL}+$ [30], which provide efficient algorithms for classifying big terminologies. Recently, Stoilos et. al. [31] presented an algorithm for f_G-$\mathcal{EL}+$ which classifies terminologies that also use fuzzy subsumption [25]. An overview of the field of fuzzy Description Logics can also be found in [32].

As we see from the above, regarding the theoretical side, fuzzy DLs have been studied relatively enough and their logical and mathematical properties are beginning to get quite understood. Another important side is the development

[5] `http://dipper.csd.abdn.ac.uk/OntoSearch/`

of tools and systems that would provide a flexible and efficient way to build and manage fuzzy knowledge. Although this aspect has not been explored much yet, there are again some first works towards this direction. We have reported about one such work in the current paper, and more precisely the FiRE system, which consists of (i) a beta fuzzy DL reasoner for f_{KD}-$\mathcal{SHIN}$, (ii) a GUI for editing and creating fuzzy KBs using the KRSS format and (iii) a module that provides persistent storage of large amounts of fuzzy knowledge bases in the RDF triple store Sesame and implements very expressive fuzzy conjunctive queries [11] over it, by extending Sesame's SeRQL query.

Still there is plenty of way to go until we can provide adequate support for fuzzy knowledge engineering and management. First, no support for parsing RDF/XML files that contain fuzzy assertions (as these have been described in [7]) exists. Moreover, there is currently no evidence about the scalability of the existing expressive fuzzy DL reasoning systems. In other words optimization techniques need to be investigated; some preliminary investigations have been carried out in [33] but still no evaluation or fuzzy DL system has been reported. Most important of all, besides the very basic manual support provided by current systems, there are currently no available graphical tools for assisting end users to (semi) automatically create fuzzy knowledge bases from raw numerical data. All these issues are very important in order for fuzzy DL technologies to be more widely adoptable in the Semantic Web.

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